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(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2021 ONLY)

21PMS312

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS :DECEMBER - 2022

COURSE NAME: M.Sc.- MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: III **TIME : 3 HOURS**

FUNCTIONAL ANALYSIS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. Every finite dimensional subspace of a normed linear space is _____________.

   a) open b) closed c) convex d) complete

2. Two projections P and Q are orthogonal if ___________.

   a) $PQ = 1$ b) $PQ = 2$ c) $PQ = 0$ d) $PQ \neq 1$

3. A linear operator $A: X \to Y$ is a linear isometry if and only if _______________.

   a) $||A(x)|| \leq c||x|| \; \forall x \in X$ b) $||A(x)|| = c||x|| \; \forall x \in X$
   c) $||A(x)|| = ||x|| \; \forall x \in X$ d) $||A(x)|| \leq ||x|| \; \forall x \in X$

4. If X be a normed linear space and $x \in X$ then there exist $f \in X'$ such that _____.

   a)$f(x) = ||x||, ||f|| = 1$ b) $f(x) \neq ||x||$ c) $f(x) \leq ||x||, ||f|| = 1$ d) $f(x) \geq ||x||$

5. Let X and Y be normed linear spaces and $A: X \to Y$ be a linear operator, if A is an open map then A is ______.

   a) injective b) surjective c) bijective d) continuous

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define normed linear space.
7. Define convex set.
8. Write the Parseval's formula.
9. Define reflexive space.
10. State Arzela-Ascoli theorem.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) State & prove Minkowski's Inequality in $C[a, b]$.

(OR)

b) State & prove Polarization identities.

(CONTD……2)

12. a) State and prove Riesz lemma.

(OR)

b) State and prove the Projection Theorem.

13.a) Let X and Y be normed linear spaces. If X is finite dimensional prove that every linear operator $A: X \to Y$ is continuous.

(OR)

b) If X is a Hilbert space then prove that for every continuous functional f on X, there exists a unique $v \in X$ such that $f(x) = < x, v > \ \forall x \in X$.

14.a) Let X and Y be normed linear spaces such that $X \neq \{0\}$ and let $B(X, Y)$ be a Banach space then prove that Y is a Banach space.

(OR)

b) Let X and Y be a normed linear spaces and $A \in \mathcal{B}(X, Y)$ prove that $||A'|| = ||A||$.

15.a) State and prove Uniform Boundedness theorem.

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(OR)

b) State & prove Open mapping theorem.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS.
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. State and prove Baire- Category Theorem.

17.i) Define Banach space.

ii) Let Y be a closed subspace and Z be a finite dimensional subspace of a normed linear space X prove that $Y + Z$ is a closed subspace of X.

18. State and prove Heine-Borel theorem.

19. i) State and prove Bessel's Inequality.

ii) If X is a Hilbert space and E is an orthonormal basis of X then prove that E is countable if and only if X is countable.

20. State & prove Hahn Banach extension Theorem.

21. State and prove Closed graph Theorem.

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ETHICAL PAPER