(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS104

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : DECEMBER-2022

COURSE NAME : M.Sc.-MATHEMATICS MAXIMUM MARKS: 50

SEMESTER: I TIME : 3 HOURS

ORDINARY DIFFERENTIAL EQUATIONS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1_____ system is called a linear homogeneous system of the n th order.

(a) $x' = A(t)x \quad (t \in I)$ (b) $x^n = A(t)x \quad (t \in I)$

(c) $x' = A(t)nx^{n-1} \quad (t \in I)$ (d) $x' = A^n(t)x \quad (t \in I)$

2. ___ is called a Bessel function of order n of the first kind.

(a) J_y (b) $J_x(n)$ (c) $J_n(x)$ (d) $J_0(x)$

3. ____ is a regular singular point for $L(Yy) = y'' + a_1(x)y' + a_2(x)y = 0$

(a) 0 (b) 1 (c) ∞ (d) finite

4. A zero of a non trivial solution of $L(x) = (p(t)x')' + g(t)x = 0, \ (a < t < b)$ is _________.

(a) Converges (b) diverges (c) isolated (d) oscillated

5. $U_i(\alpha x_1 + \beta x_2) = \alpha U_i x_i + \beta U_i x_2$. This linear operator U_i is called ____

(a) Linearly independent (b) Green's formula

(c) Boundary form (d) complementary boundary form

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Find the solution of homogeneous system $x' = Ax$ if n=1.
7. Define singularities.
8. Define non homogeneous linear systems.
9. Write any one fundamental result.
10. Write the adjoint boundary condition.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that if ϕ_1 and ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0, then $W(\phi_1, \phi_2)(x) = e^{-a_1(x - x_0)} \, W(\phi_1, \phi_2)(x_0)$.

(OR)

b) Solve $y'' = 3y' + 3y = 0$

12. a) Solve $x^2 \, y'' + a(x)x\,y' + b(x)y = 0$ where a,b have convergent series expansions for $|x| < r_0$

(OR)

b) Derive the indicial Polynomial for $L(y) = x^2 \, y'' + \frac{3}{2} x\,y' + xy = 0$

(CONTD……2)

ETHICAL PAPER

13.a) Show that the equations the equation $y'=\frac{3x^2-2xy}{x^2-2y}$ is exact and solve it.

(OR)

b) Provw that ϕ is a real-valued solution of $h(y)\frac{dy}{dx}=g(x)$ on the interval I containing a point x_0 iff $\int_{\phi(x_0)}^{\phi(x)} h(u)\,du=\int_{x_0}^{x} g(t)\ dt$

14.a) Prove that if $\phi_1,\phi_2,\ldots\ldots\ldots\ldots\phi_n$ be n linearly independent solutions of L(y)=0 on an interval I. and $c_1,c_2,\ldots\ldots\ldots\ldots c_n$ are any constants, then $\phi=c_1\phi_1+c_2\phi_2+\ldots\ldots\ldots\ldots+c_n\phi_n$ is a solution.

(OR)

b) Let $\phi_1,\phi_2,\ldots\ldots\ldots\ldots\phi_n$ be n solutions of L(y)=0 on an interval I satisfying $\phi_i^{(i-1)}(x_0)=1,\ \phi_i^{(i-1)}(x_0)=0\ \ ,\ \ j\neq i$. If ϕ is any solution of L(y)=0 on I, Prove that there are n constants $c_1,c_2,\ldots\ldots\ldots\ldots c_n$ such that $\phi=c_1\phi_1+c_2\phi_2+\ldots\ldots\ldots\ldots+c_n\phi_n$

15.a) Derive Green's formula.

(OR)

b) Derive Boundary from formula.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Prove that two solutions ϕ_1 and ϕ_2 of L(y)=o are linearly independent on an interval I , iff $W(\phi_1,\phi_2)(x)\neq 0$ for all x in I.

(OR)

(b) Let ϕ be any solution of $L(y)=y''+a_1y'+a_2\ y=0$ on an interval I containing a point x_0 . Then prove that for all x in I, $\|\phi(x_0)\|\left|e^{-k|x-x_0|}\right|\leq\|\phi(x)\|\leq\|\phi(x_0)\|e^{k|x-x_0|}$ where

$$\|\phi(x)\|=[|\phi(x)|^2+|\phi'(x)|^2]^{\frac{1}{2}} \qquad ,k=1+|a_1|+|a_2|$$

17. a) Derive the Bessel function of zero order of the first kind.

(OR)

b) Evaluated the basis for the Legendre equation $L(y)=(1-x^2)y''-2xy'+\alpha(\alpha+1)y=0$ where α is a constant.

18. a) State and prove Existence theorem.

(OR)

b) State and prove uniqueness theorem.

19. a) State and prove strum's comparison theorem.

(OR)

b) State and prove comparison theorem of Hille-Winter.

20. a) State and prove Strum-Liouville's problem.

(OR)

b) State and prove Picard theorem.

****

ETHICAL PAPER