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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022ONLY)

22PMS102

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : DECEMBER - 2022

COURSE NAME: M.Sc.- MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: I **TIME : 3 HOURS**

REAL ANALYSIS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. If $\alpha \nearrow on\ [a,b]$, $P_1\ and\ P_2$ are any two partitions then _______.
   a) $L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$ b) $L(P_1, f, \alpha) \geq U(P_2, f, \alpha)$
   c) $L(P_1, f, \alpha) = U(P_2, f, \alpha)$ d) $L(P_1, f, \alpha) < U(P_2, f, \alpha)$
2. The sequence f function $\{f_n\}$ where $f_n(n) = \frac{x}{1+nx}$, $x \in [0, 08)$ converges.
   a) pointwise to 1 b) pointwise to 0 c) uniformly to 0 d) uniformly to 1
3. The simplest form of an equation of an implicit function theorem is _______.
   a) $f(x,c) = 0$ b) $f(x,x_0) = 0$ c) $f(x,t) = 0$ d) $f(x) = 0$
4. A ______ function defined on an interval is measurable.
   a) monotone b) bounded c) continuous d) characteristic
5. If f is defined on [0,1] by setting $f(x) = 1$ if x is rational and 0 if x is irrational then f is ____________.
   a) Riemann integrable b) not Riemann integrable c) measurable d) continuous

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define Rectifiable curves.
7. Define equicontinuous.
8. Prove that for an linear operator which is invertible, $\det[A] \neq 0$.
9. Define Lebesgue measure.
10. State Monotone Convergence Theorem.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If f is monotonic on $[a,b]$ and α is continuous on $[a,b]$ then prove that $f \in R(\alpha)$.

(OR)

b) State and prove fundamental theorem of calculus.

12. a) State and prove Cauchy criterion for uniform convergence.

(OR)

b) Prove that if K is a compact metric space if $f_n \in \mathcal{C}(k)\ for\ n = 1,2,3 \ldots$, and if $\{f_n\}$ converges uniformly on K then $\{f_n\}$is equicontinuous on K.

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13.a) Prove that a linear operator A on a finite-dimensional vector space X is one-one if and only if the range of A is all of X.

(OR)

b) If $[A]$ and $[B]$ are n X n matrices then prove that $det([B][A]) = \det[B]\det[A]$.

14.a) Prove that the union of a finite collection of measurable sets is measurable.

(OR)

b) Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise a.e on E to the function f prove that f is measurable.

15. a) If f is a bounded measurable function on a set of finite measure E, prove that f is integrable over E.

(OR)

b) State and prove Chebychev's Inequality.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Prove that if P^* is a refinement of P, then $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(P, f, \alpha)$.

(OR)

b) Let $f \in R\ on\ [a, b].\ for\ a \leq x \leq b,\ put\ F(x) = \int_a^x f(t)dt$,then prove that F is continuous on [a,b] and also prove that if f is continuous at a point $x_0\ then\ F'(x_0) = f(x_0)$.

17. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim_{t\to x} f_n(t) = A_n$ $(n = 1,2,3 \dots .)$ Prove that $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

(OR)

b) State and prove The Stone-Weisrstrass Theorem.

18. a) Prove that if X is a complete metric space and if φ is a contraction of X into X then there exists one and only one $x \in X$ such that $\varphi(x) = x$.

(OR)

b) State and prove the Inverse function theorem.

19. a) Prove that the outer measure of an interval is its length.

(OR)

b) State and prove the Simple Approximation Theorem.

20. a) State and prove Bounded Convergence Theorem.

(OR)

b) State and prove Fatou's Lemma.

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