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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

20UMS509

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : DECEMBER-2022

COURSE NAME: B.Sc-MATHEMATICS (Aided & SF) **MAXIMUM MARKS: 70**

SEMESTER: V **TIME : 3 HOURS**

PART – III

MODERN ALGEBRA

SECTION -A ($\mathbf{10 \times 1 = 10}$ **MARKS**)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **K1**

1. If G is a group such that $(a.b)^2 = a^2.b^2$ for all $a, b \in G$, then G must be __________.
   a) Finite group b) subgroup c) abelian d) closed
2. A homomorphism ϕ $from\ G\ into\ \bar{G}$ is said to be _________ if $\phi\ is\ one - to - one$.
   a) A kernel b) an abelian c) a inverse image of G d) isomorphism
3. _________ is an equivalence relation on G.
   a) Conjugacy b) Normalizer c) Partitions d) Normal subgroup
4. The homomorphism $\phi\ of\ R\ into\ R'$ is an isomorphism if and only if ___________.
   a) $I(0) = \phi$ b) $I(\phi) = (0)$ c) $R(I) = \phi$ d)$R(\phi) = I$
5. Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not unit in R, then ____________.
   a) $d(a) > d(ab)$ b) $d(a) \leq d(ab)$ c) $d(a) = d(ab)$ d) $d(a) < d(ab)$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. K2

6. Define an abelian group.
7. Define normal subgroup.
8. Define normalizer of $a$in G.
9. Define Zero divisor.
10. Define Euclidean ring.

SECTION-B ($\mathbf{5 \times 4 = 20}$ **MARKS**)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWNG QUESTIONS. K3

11. a) Show that if every element of the group G is its own inverse, then G is abelian.

(OR)

   b) Prove that HK is a subgroup of G if and only if $HK = KH$.

(CONTD.....2)

ETHICAL PAPER

12. a) Prove that the subgroup $N\ of\ G$ is a normal subgroup of G if and only ifevery left coset of N is$G$ is a right coset of $N\ is\ G.$

(OR)

b) If ϕ is a homomorphism of $G\ into\ \bar{G}$ with kernel K, such that K is a normal subgroup of G.

13. a) Prove that every permutation is the product of its cycles.

(OR)

b) Show that if $O(G) = P^n where\ P\ is\ a\ prime\ number, Z(G) \neq (e)$.

14. a) What is definition of a ring? Give an example.

(OR)

b) If $\phi$is a homomorphism of $R\ into\ R'$ with kernel $I(\phi)$, show that (i) $I(\phi)$ is a subgroup of R under addition. (ii) If $a \in I(\phi)$ and $r \in R$ then both $ar\ and\ ra\ are\ in\ I(\phi)$.

15. a) Let R be a commutative ring with unit element whose only ideals are $(0) and\ R$ itself. Show that R in a field.

(OR)

b) State and Prove Fermat theorem.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS
(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Prove that every integral domain can be imbedded is a field.

17. If $H\ and\ K$ are finite subgroups of G of orders $O(H)\ and\ O(K)$respectively. Show that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

18. State and prove Cauchy's theorem for abelian groups.

19. Prove that every group is isomorphic to a subgroup of $A(s)$ for some appropriate S.

20. Prove that a finite integral domain is a field.

21. If R is a commutative ring with unit element and M is an ideal of R, prove that M is a maximal ideal of R if and only if R/M is a field.

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