

(FOR THE CANDIDATES ADMITTED

20UMS512 / 20UMA512

DURING THE ACADEMIC YEAR 2020 ONLY)

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS: DECEMBER- 2022
COURSE NAME: B.Sc.- MATHEMATICS (Aided & S.f) MAXIMUM MARKS: 70
SEMESTER: V TIME : 3 HOURS

PART - III
THEORY OF NUMBERS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.**MULTIPLE CHOICE QUESTIONS (K1)**

- Use the mathematical induction to find the formula for all $n \geq 1$,
 $1(1!)+2(2!)+3(3!)+.....+n(n!)=?$
a) $(n+1)!$ b) $n!$ c) $(n-1)!$ d) $(n+1)!-1$
- Congruence integer's are_____.
(a) Equal (b) not necessarily equal (c) primes (d) zero integers
- Find the solution of the system of congruence's $7x+3y \equiv 10 \pmod{16}$ and $2x+5y \equiv 9 \pmod{16}$
a) 3 & 7 b) 9 & 6 c) 1 & 2 d) 4 & 5
- $\sigma(180) =$ _____.
a) 546 b) 350 c) 250 d) 35
- If $(a, p) = 1$ and $x^n \equiv a \pmod{p}$ has a solution then a is called_____
(a) p^{th} power residue modulo p (b) n^{th} power residue modulo p
(c) p^{th} power residue modulo n (d) n^{th} power residue modulo n

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

- If a and b are relatively prime positive integers, then the Diophantine equation $ax-by=c$ has how many solutions in the positive Integers.
- Write the statement of Fermat's little's theorem.
- The linear congruence $9x \equiv 21 \pmod{30}$ has how many number of solution?
- For $N=6$, Find $\sum \sigma(n) = ?$
- Define primitive .

SECTION - B (5 X 4 = 20 MARKS)**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**

- a) Prove that the sum of n positive integers is $\frac{n(n+1)}{2}$.

(OR)

- b) State and prove fundamental theorem of arithmetic.
- a) Prove that the product of any n consecutive positive inters is divisible by the product of the first n positive integers.

(OR)

- b) If s integer $r_1, r_2, \dots, r_s$ form a reduced residue system modulo m then $s = \phi(m)$.

13. a) State and prove Chinese remainder theorem.

(OR)

b) For arbitrary integer a and b , $a \equiv b \pmod{n}$ iff a and b leave the same nonnegative remainder when divided by n .

14. a) Prove that τ and σ are both multiplicative functions.

(OR)

b) Prove that if f is multiplicative function and F is defined by $F(n) = \sum_{d|n} f(d)$, then F is also multiplicative and $F(8.3) = F(8)F(3)$.

15. a) If p is a prime, then prove that there exist $\phi(p-1)$ primitive roots modulo p .

(OR)

b) State and prove Euler's criterion.

SECTION - C

(4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS .

(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS
(FROM Qn. No : 17 to 21). (K4 (Or) K5)

16. State and prove Euclid division algorithm.

17. Prove that $a_n = \binom{2n-2}{n-1} / n$

18. Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution iff $p \equiv 1 \pmod{4}$

19. Let $n > 0$ be fixed and a, b, c, d be arbitrary integer, Then prove that the following

(i) $a \equiv a \pmod{n}$

(ii) If $a \equiv b \pmod{n}$ then $b \equiv a \pmod{n}$

(iii) If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$

(iv) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $a + c \equiv b + d \pmod{n}$ and $ac \equiv bd \pmod{n}$

(v) If $a \equiv b \pmod{n}$ then $a + c \equiv b + c \pmod{n}$ and $ac \equiv bc \pmod{n}$

20. State and prove Mobius inversion theorem.

21. State and prove Tchebyshev's Theorem.
