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**(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2020 ONLY)**

**20UMS512 / 20UMA512**

**REG.NO. :**

**N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI**
**END-OF-SEMESTER EXAMINATIONS: DECEMBER- 2022**
**COURSE NAME: B.Sc.- MATHEMATICS (Aided & S.f )** **MAXIMUM MARKS: 70**
**SEMESTER: V** **TIME : 3 HOURS**

## PART - III

## THEORY OF NUMBERS

### SECTION - A (10 X 1 = 10 MARKS)

**ANSWER THE FOLLOWING QUESTIONS.**
**MULTIPLE CHOICE QUESTIONS (K1)**

1. Use the mathematical induction to find the formula for all $n \geq 1$, $1(1!)+2(2!)+3(3!)+........+n(n!)=?$
   a) (n+1)! b) n! c) (n-1)! d) (n+1)!-1

2. Congruence integer's are____________.
   (a) Equal (b) not necessarily equal (c) primes (d) zero integers

3. Find the solution of the system of congruence's $7x+3y \equiv 10 \pmod{16}$ and $2x+5y \equiv 9 \pmod{16}$
   a) 3 &7 b) 9&6 c) 1&2 d) 4&5

4. $\sigma(180)$ =_______________.
   a) 546 b) 350 c) 250 d) 35

5. If $(a, p)=1$ and $x^n \equiv a \pmod{p}$ has a solution then a is called__________
   (a) $p^{th}$ power residue modulo p (b) $n^{th}$ power residue modulo p
   (c) $p^{th}$ power residue modulo n (d) $n^{th}$ power residue modulo n

**ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)**

6. If a and b are relatively prime positive integers, then the Diophantine equation ax-by =c has how many solutions in the positive Integers.
7. Write the statement of Fermat's little's theorem.
8. The linear congruence $9x \equiv 21 \pmod{30}$ has how many number of solution?
9. For N=6, Find $\sum \sigma(n)$ =?
10. Define primitive .

### SECTION – B (5 X 4 = 20 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**

11. a) Prove that the sum of n positive integers is $\frac{n(n+1)}{2}$.

**(OR)**

b) State and prove fundamental theorem of arithmetic.

12. a) Prove that the product of any n consecutive positive inters is divisible by the product of the first n positive integers.

**(OR)**

b) If $s$ integer $r_1, r_2, ..................r_s$ form a reduced residue system modulo m then $s = \phi(m)$.

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**(CONTD…….2)**

13. a) State and prove Chinese remainder theorem.

**(OR)**

b) For arbitrary integer a and b , $a \equiv b(\bmod\ n)$ iff a and b leave the same nonnegative remainder when divided by n.

14. a) Prove that $\tau$ and $\sigma$ are both multiplicative functions.

**(OR)**

b) Prove that if $f$ is multiplicative function and $F$ is defined by $F(n) = \sum_{d/n} f(d)$ ,then $F$ is also multiplicative and $F(8.3) = F(8)F(3)$ .

15. a) If p is a prime, then prove that there exist $\phi(p-1)$ primitive roots modulo p.

**(OR)**

b) State and prove Euler's criterion.

**SECTION - C (4 X 10 = 40 MARKS)**

**ANSWER ANY FOUR OUT OF SIX QUESTIONS .**
**(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21). (K4 (Or) K5)**

16. State and prove Euclid division algorithm.

17. Prove that $a_n = \binom{2n-2}{n-1}/n$

18. Prove that the quadratic congruence $x^2 + 1 \equiv 0(\bmod p)$, where p is an odd prime, has a solution iff $p \equiv 1(\bmod\ 4)$

19. Let $n > 0$ be fixed and $a, b, c, d$ be arbitrary integer, Then prove that the following
   (i) $a \equiv a(\bmod n)$
   (ii) If $a \equiv b(\bmod n)$ *then* $b \equiv a(\bmod n)$
   (iii) If $a \equiv b(\bmod n)$ *and* $b \equiv c\ (\bmod n)$ *then* $a \equiv c\ (\bmod\ n)$
   (iv) If $a \equiv b\ (\bmod\ n)$ *and* $c \equiv d\ (\bmod\ n)$ *then* $a + c \equiv b + d(\bmod n)$ *and* $ac = bd(\bmod\ n)$
   (v) If $a \equiv b \bmod n)$ *then* $a + c \equiv b + c\ (\bmod\ n)$ *and* $ac \equiv bc\ (\bmod\ n)$

20. State and prove Mobius inversion theorem.

21. State and prove Tchebyshev's Theorem.

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