(FOR THE CANDIDATES ADMITTED

 DURING THE ACADEMIC YEAR 2022 ONLY)

22UMS101

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : DECEMBER-2022
COURSE NAME : B.Sc.-MATHEMATICS (AIDED)   MAXIMUM MARKS: 50
SEMESTER:I   TIME : 3 HOURS

PART - III

CLASSICAL ALGEBRA

SECTION – A   (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

MULTIPLE CHOICE QUESTIONS.

1.   One of the roots of the equation $x^3 - 2x - 5 = 0$ lies between_____.
   (a) 2 and 3   (b) -4 and 0   (c) 3 and 4   (d) 0 and 1

2. $(1 - x)^n =$ _____
   (a) $1 - nx + \dfrac{n(n-1)}{2!}x^2 - \dfrac{n(n-1)(n-2)}{3!}x^3 + \ldots$ (b) $1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \ldots$

   (c) $1 - nx + \dfrac{n(n+1)}{2!}x^2 - \dfrac{n(n+1)(n+2)}{3!}x^3 + \ldots$ (d) $1 + nx + \dfrac{n(n+1)}{2!}x^2 + \dfrac{n(n+1)(n+2)}{3!}x^3 + \ldots$

3.   If α, β, γ are the roots of the equation $81x^3 - 18x^2 - 36x + 8 = 0$, then $\sum \alpha =$ _____ .

   (a) $\dfrac{18}{81}$ (b) $\dfrac{-36}{81}$ (c) $\dfrac{-8}{81}$ (d) $-\dfrac{18}{81}$

4.   The difference between the number of changes of sign will give positive roots
   when _____are substituted in the series of Strum's function.
      (a) $0\ \&\ \infty$ (b) $0\ \&\ -\infty$ (c) $-\infty\ \&\ \infty$ (d)   0

5.   If T is a nilpotent linear operator on a vector space of dimension n , then the characteristic
   polynomial for T is_____.
      (a) n   (b) T^n (c) x^n (d) $T^n = 0$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.   (K2)

6.   Write the methods for obtaining approximate solutions for algebraic equations
7.   Write the value of   in commensurable number
8.   Find the quotient and the remainder when $2x^6 + 3x^5 - 15x^2 + 2x - 4$ is divided by $x + 5$.
9.   In which method can be used to determine both commensurable and in commensurable
   roots of a numerical equation?
10.   Define orthogonal matrices.

SECTION – B   (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11(a).   Perform four iterations of the bisection method to obtain the smallest positive root of
      the equation $f(x) = x^3 - 5x + 1 = 0$

(OR)

   (b) Find the smallest positive root of $x^2 - \log_e^x - 12 = 0$ by Regula-Falsi method

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12.(a) Determine the coefficient of x^n in the expansion of $\dfrac{x+1}{(x-1)^2(x-2)}$

(OR)

(b) Sum the series to infinity $\dfrac{1.4}{5.10} - \dfrac{1.4.7}{5.10.15} + \dfrac{1.4.7.10}{5.10.15.20} \ldots\ldots\ldots\ldots$

13.a) Find the transformed equation, Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$

(OR)

b) If $a+b+c+d = 0$ show that $\dfrac{a^5+b^5+c^5+d^5}{5} = \dfrac{a^2+b^2+c^2+d^2}{2} \cdot \dfrac{a^3+b^3+c^3+d^3}{3}$

14 (a) Find the number of real roots of the equation $x^4 - 14x^2 + 16x + 9 = 0$

(OR)

(b) Find the multiple roots of the equation $x^4 - 9x^2 + 4x + 12 = 0$

15.a) Find the characteristic roots of the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

(OR)

b) Define idempotent matrix and show that $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ -1 & -3 & -4 \end{bmatrix}$ is idempotent matrix.

SECTION – C   **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Find a real root of the equation $x^3 - 3x + 1 = 0$ lying between 1 and 2 correct to three places of decimal by using bisection method.

(OR)

b) Use the method of iteration to solve the equation 2x= cos x+3.

17.a) Evaluate the sum to infinity of the series.

$\dfrac{1}{24} - \dfrac{1.3}{24.32} + \dfrac{1.3.5}{24.32.40} - \ldots\ldots\ldots\ldots$

(OR)

b) (i) Show that $\log \sqrt{12} = 1 + \left(\dfrac{1}{2} + \dfrac{1}{3}\right)\dfrac{1}{4} + \left(\dfrac{1}{4} + \dfrac{1}{5}\right)\dfrac{1}{4^2} + \left(\dfrac{1}{6} + \dfrac{1}{7}\right)\dfrac{1}{4^3} + \ldots\ldots\ldots$

(ii) Sum the series $1 + \dfrac{1+3}{2\,!} + \dfrac{1+3+3^2}{3\,!} + \dfrac{1+3+3^2+3^3}{4\,!} + \ldots\ldots\ldots\ldots\infty$

18.a) If $\alpha, \beta, \gamma, \delta$ are the roots of biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$ find (i) $\sum \alpha^2$ (ii) $\sum \alpha^2 \beta\gamma$ (iii) $\sum \alpha^2 \beta^2$ (iv) $\sum \alpha^3 \beta$ (v) $\sum \alpha^4$

(OR)

b) Solve the equation $x^4 + 20x^3 - 143x^2 + 430x + 462 = 0$ by removing its second term.

19.a) The equation $x^3 - 3x + 1 = 0$ has a root between 1 and 2 . Calculate it to three places of decimals.

(OR)

b) Use Horner's method to solve $2x^3 + 3x^2 + 4x + 5 = 0$.

20.a) State and prove Cayley Hamiltonian Theorem.

(OR)

b) Find the Eigen values and Eigen vectors of the matrix A= $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

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