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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS207

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY-2025

M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: II **TIME : 3 HOURS**

PART - III
PARTIAL DIFFERENTIAL EQUATIONS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. Eliminate the constants a and b from $z = (x + a)(y + b)$_____.

   (a) $z = pq$ (b) $z = (x + p)(y + q)$ (c) $z = xy$ (d) $z = p + q$

2. If $u = f(x + iy) + g(x - iy)$, where the functions f and g are arbitrary functions, then__

   (a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2$ (b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = xy$ (c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = xf''(x + iy)$

3. The kernel $K(\xi, x)$ in the Fourier transform of $f(x)$ is _____.

   (a) $\frac{1}{\sqrt{2\pi}} e^{i\xi x}$ (b) $e^{i\xi x}$ (c) $e^{-i\xi x}$ (d) $\sqrt{e^{i\xi x}}$

4. If $\rho > 0$ and $\psi(r) = \int_V \frac{\rho(\boldsymbol{r}')d\tau'}{|\boldsymbol{r}-\boldsymbol{r}'|}$, where the volume V is bounded, then $\lim_{r\to\infty} r\,\psi(r)$ is_____,

   (a) $\int_V \psi(\boldsymbol{r}')d\tau'$ (b) $\int_V \rho(\boldsymbol{r}')d\tau'$ (c) $\psi \int_V \rho(\boldsymbol{r}')d\tau'$ (d) 0

5. A general solution of the wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ is_____.

   (a) $y = f(x + ct) + g(x + ct)$ (c) $y = f(x + ct) + g(c + xt)$

   (b) $y = f(x + ct) + g(c - xt)$ (d) $y = f(x + ct) + g(x - ct)$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Eliminate the arbitrary function f from the equation $z = xy + f(x^2 + y^2)$.
7. Classify the second order PDE of the type $Rr + Ss + Tt + f(x, y, z, p, q) = 0$.
8. Name any one method to solve nonlinear equations of the second order?
9. Define the equipotential surfaces.
10. Write down the two-dimensional diffusion equation.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Show that the equations $xp = yq$ and $z(px + qy) = 2xy$ are compatible and solve them.

(OR)

b) Find a complete integral of the equation $(p + q)(z - xp - yq) = 1$.

(CONTD……2)

12. a) Find a particular integral of the equation $(D^2 - D')z = 2y - x^2$.

(OR)

b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

13. a) Solve the one-dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ by using separation of variables.

(OR)

b) Solve the equation $q^2 r - 2pqs + p^2 t = 0$.

14. a) Prove that $r\cos\theta$ and $r^{-2}\cos\theta$ satisfy the Laplace equation, when $r, \theta, \emptyset$ are spherical polar coordinates.

(OR)

b) Discuss the two main types of Boundary Value Problem for Laplace equations.

15. a) The points of trisection of a string are pulled aside through a distance ϵ on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

(OR)

b) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and it initial temperature is $f(r, \theta)$.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Find a complete integral, general integral and singular integral of the equation $z^2(1 + p^2 + q^2) = 1$.

(OR)

b) Find a complete integral of the equation $p^2 x + q^2 y = z$ by using Charpit's method.

17. a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

(OR)

b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

18. a) Derive the solution of the equation: $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$ for the region $r \geq 0, z \geq 0$, satisfying the conditions: (i) $V \to 0$ as $z \to \infty$ and as $r \to \infty$ (ii) $V = f(r)$ on $z = 0, r \geq 0$.

(OR)

b) Solve the equation $r + 4s + t + rt - s^2 = 2$.

19. (a) Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

(OR)

(b) A uniform insulated sphere of dielectric constant κ and radius a carries on its surface a charge density $\lambda P_n(\cos\theta)$, show that the interior of the sphere contributes an amount $\frac{8\pi^2\lambda^2 a^3 \kappa n}{(2n+1)(\kappa n+n+1)^2}$ to the electrostatic energy.

20. (a) A thin membrane of great extent is released from rest in the position $z = f(x, y)$. Determine the displacement at any subsequent time.

(OR)

(b) The faces $x = 0, x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $\theta = f(x)(0 < x < a)$. Determine the temperature at a subsequent time t.

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