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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS417

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : MAY - 2025
M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: IV **TIME : 3 HOURS**

ALGEBRAIC TOPOLOGY

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. If $f \approx f^{`}$ and $f^{`}$ is a constant map, then *f is called* _________.
   (a) Final point of the path (b) homotopic (c) nulhomotopic (d) path homotopic

2. A group is cyclic of infinite order iff it is isomorphic to the _____ group of integers.
   (a) $\frac{z}{k}$ (b) multiplicative (c) additive (d) divisible

3. Two spaces that are homotopy equivalent are said to have the _____ type
   (a) Deformation retract (b) same homotopy (c) nulhomotopy (d) homotopy inverse

4. A simplex in K is said to be______ if it is not contained in another larger simplex.
   (a) Maximal (b) minimal (c) pure simplicial complex (d) isomorphism

5. _____ is defined for simplicial complexes, which are the & complexes whose simplices are uniquely determined by their vertices
   (a) Chain complex (b) barycentric coordinates (c) homology classes (d) simplicial homology

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define simply connected.
7. Define evenly covered spaces.
8. Write the statement of Brouwer fixed –point theorem for the disc .
9. What is called is called a triangulable space.?
10. What is called commutative diagram?

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that $\approx$ and $\approx_{p^{`}}$ are equivalence relations.

(OR)

b) Prove that the map $\hat{\alpha}$ is a group isomorphism.

12. a) Prove that the fundamental group of S’ is isomorphic to the additive group of integers

(OR)

b) If $p : E \to B$ and $p' : E' \to B'$ are covering map then Prove that $p \times p' = E \times E' \to B \times B'$ is a covering map.

13. a) Prove that given a non vanishing vector field on B^2, there exists a point of S’ where the vector field points directly inward and a point of S’ where its points directly outward

(OR)

b) Prove that the inclusion $j : S^n \to S^{n+1} - 0$ induces an isomorphism of fundamental group.

(CONTD…….2)

14.a) Let X ⊂ R n be compact. Prove that a point x ∈ X is a relative boundary point of X iff there exist arbitrarily small neighborhoods U of x in X such that every continuous function f : X \ U → S n−1 has a continuous extension over X.

(OR)

b) Prove that any triangulation (K, f) of a topological space X defines a regular CW-structure on X such that the n th-skeleton is given by X(n) = f(|K(n) |), n ≥ 0.

15.a) Prove that the composition $\Delta_n(x) \xrightarrow{\partial n} \Delta_{n-1}(X) \xrightarrow{\partial_{n-1}} \Delta_{n-2}(X)$ is zero.

(OR)

b) For good pairs (X, A), prove that the quotient map $q:(X,A) \to (X/A, A/A)$ induces isomorphisms $q^*: H_n(X,A) \to H_n(X \mid A, A \mid A) \approx \tilde{H}n(X \mid A) \quad \forall n$

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Prove that operation * has Associatively, inverse ,Right and left identities properties

(OR)

b) Prove that

(i) If $h:(X,x_0) \to (Y,y_0)$ and $k:(Y,y_0) \to (Z,z_0)$ are continuous then $(k \circ h)_* = k_* \circ h_*$ *if* $i:(X,x_0) \to (X,x_0)$ is the identity map then i_* is the identity homomorphism.

(ii) Prove that In a simply connected space X, any two paths having the same initial and final points are path homotopic .

17. a) Prove that the map $p: R \to S'$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map

(OR)

b) Let $p;E \to B$ be a covering map, let $p(e_0) = b_0$ Prove that any path $f:[0,1] \to B$ beginning at b_0 has a unique lifting to a path $\tilde{f}$ in E beginning at e_0

18. a) State and prove fundamental theorem of algebra

(OR)

b) Let $F: X \to Y$ be continuous let $f(x_0) = y_0$. If f is a homotopy equivalence then prove that $f^*: \pi_1(X,x_0) \to \pi_1(Y,y_0)$ is an isomorphism.

19.a) Let K be a finite simplicial complex and d be a linear metric on |K|. Then prove that d is a linear metric on |sd K| = |K| and for any F ′ ∈ sd K such that F ′ ⊂ |F|, where F ∈ K is a q-simplex, we have the inequality of the diameters: diam|F ′ | ≤ q q + 1 diam|F|.

(OR)

b) State and prove Sperner's lemma.

20.a) If X is nonempty and path-connected, then $H_0(X) \approx Z$ Prove that for any space X , H_0 (X) is a direct sum of Z's, one for each path-component of X .

(OR)

b) If two maps $f, g: X \to Y$ are homotopic , then prove that they induce the same homomorphism $f^* = g^*: H_n(X) \to H_n(Y)$

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