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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS4E1

REG.NO.:

N.G.M. COLLEGE (AUTONOMOUS): POLLACHI
END-OF-SEMESTER EXAMINATIONS: MAY-2025
M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: IV **TIME: 3 HOURS**

MATHEMATICAL METHODS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. The resolvent kernel for the integral equation $g(s) = f(s) + \lambda \int_{-1}^{1} (st + s^2t^2)\, g(t)dt$ is _____.
   a. $\frac{st}{1-(2\lambda/3)} - \frac{s^2t^2}{1-(2\lambda/5)}$ b. $\frac{st}{1-(2\lambda/3)} + \frac{s^2t^2}{1-(2\lambda/5)}$
   c. $\frac{st}{1+(2\lambda/3)} + \frac{s^2t^2}{1-(2\lambda/5)}$ d. $\frac{st}{1-(2\lambda/3)} + \frac{s^2t^2}{1+(2\lambda/5)}$
2. Initial value problem is reduced to ---------- integral equation with given initial conditions.
   a. Fredholm Type b. Volterra Type c. both d. none of these
3. A linear functional is a functional that satisfies the following condition:
   a. $l[cx] = lc(x)$ b. $l[cx] = cl(x)$
   c. $L[cy(x)] = Lc[y(x)]$ d. $L[cy(x)] = cL[y(x)]$
4. The sufficient condition for a functional v to achieve a weak minimum, if _____ .
   a. $E \geq 0$ b. $E \leq 0$ c. $E > 0$ d. $E < 0$
5. _____ employed a third method of investigations into the field of the calculus of variations.
   a. Euler b. N. Krylov c. L. Kantorovich d. N.Bogolyubov

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Define singular integral equations.
7. Write the Abel's integral equation.
8. When does the functional $v[y(x)]$ reaches a maximum on a curve $y = y_0(x)$.
9. Define extremal field.
10. Name the types of problems solved by using Galerkin's method.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.
(K3)

11. a) Find the eigenvalues and eigenfunctions of the homogeneous integral equation $g(s) = \lambda \int_1^2 [st + (1/st)]g(t)dt$.

(OR)

b) Solve the Volterra equation $g(s) = 1 + \int_0^s st\, g(t)dt$.

(CONTD……..2)

12. a) Reduce the Initial value problem $y''(s) + \lambda y(s) = F(s)$ with the initial conditions $y(0) = 1, y'(0) = 0$ to a volterra integral equation.

(OR)

b) Solve the Abel's integral equation $f(s) = \int_0^s \frac{g(t)}{(s-t)^\alpha}\, dt$,$0 < \alpha < 1$.

13. a) State and prove the fundamental theorem of calculus of variation.

(OR)

b) Find the extremals for the functional $V[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y_1^2}}{x}\, dx$.

14. a) Is the Jacobi condition fulfilled for the extremal of the functional $v = \int_0^a (y'^2 - y^2)dx$ that passes through the points $A(0,0)$ and $B(a,0)$?

(OR)

b) Test for an extremum the functional
$v[y(x)] = \int_0^a y'^2 dx; y(0) = 0, y(a) = b\,, a > 0, b > 0.$

15. a) Investigate the extremum of the functional
$v[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x} - y\right)^2 + \left(\frac{\partial z}{\partial y} + x\right)^2\right] dxdy.$

(OR)

b) Investigate for an extremum the functional
$v[z(x,y)] = \int_{-a}^{a}\int_{-b}^{b}\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 - 2z\right] dxdy;$
on the boundary of the integration domain $z = 0$.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Solve the integral equation g(s)=(e^s –s) - $\int_0^1 s(\ e^{st-1})g(t)dt$ using approximation method.

(OR)

b) Solve the integral equation $g(s) = s + \lambda \int_0^1 [st + (st)^{1/2}]g(t)dt$ using Fredholm's first theorem.

17. a) From a boundary value problem $y''(s) + A(s)y'(s) + B(s)\, y(s) = F(s)$ with $y(a) = y_0, y(b) = y_1$, reduce a Fredholm type integral equation
$y(s) = f(s) + \int k(s,t)y(t)dt.$

(OR)

b) Solve the Abel's integral equation $f(s) = \int_s^b \frac{g(t)}{[h(t)-h(s)]^\alpha}$, $0 < \alpha < 1, a < s < b$.

18. a) Derive Euler-Poission equation.

(OR)

b) Derive the differential equation of free vibrations of a string.

19. a) Test for an extremum the functional
$v[y(x)] = \int_0^{x_1} \frac{\sqrt{1+y'^2}}{\sqrt{y}}\, dx; y(0) = 0, y(x_1) = y_1\,, a > 0, b > 0.$

(CONTD……..3)

(OR)

19. b) Find the equation of geodesics on a surface on which the element of length of the curve is of the form $ds^2 = [\varphi_1(x) + \varphi_2(y)](dx^2 + dy^2)$.

20. a) Explain Euler's finite-difference method.

(OR)

b) Find a continuous solution of the equation $\Delta z = -1$ in the domain D, which is an isosceles triangle bounded by the straight lines $y = \pm\frac{\sqrt{3}}{3}x$ and $x = b$, which solution vanishes on the boundary of the domain.

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