**(FOR THE CANDIDATES ADMITTED** **22UMS615**

**DURING THE ACADEMIC YEAR 2022 ONLY)**

**REG.NO.:**

**N.G.M. COLLEGE (AUTONOMOUS): POLLACHI**
**END-OF-SEMESTER EXAMINATIONS: MAY-2025**
**B.Sc.- MATHEMATICS** **MAXIMUM MARKS: 50**
**SEMESTER: VI** **TIME: 3 HOURS**

**PART-III**
**REAL ANALYSIS-II**

**SECTION – A (10 X 1 = 10 MARKS)**
**ANSWER THE FOLLOWING QUESTIONS.**
**MULTIPLE CHOICE QUESTIONS. (K1)**

1) An infinite set has ________limit points.
   (a) Infinite (b) 2 (c) 1 (d) no
2) Let f be real on (a, b). Then f is strictly monotonically increasing on (a, b) if a<x<y<b implies.
   (a) f(x)≤f(y) (b) f(x) ≥f(y) (c) f(x) <f( y) (d) f(x)> f(y)
3) Every differentiable function is -------------------
   (a) discontinuous (b) bi-continuous (c) continuous (d) none of these
4) $\int_A^B f(x)da(x)$ *is called*______________
   (a) Riemann integral (b) complete integral (c) Riemann-Stieltjes integral (d)None of these
5) There is a real ________function on the real time which is nowhere differentiable.
   (a) Continuous (b) differentiable (c) real (d) bounded

**ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES. (K2)**

6) How to find the continuity of a composite functions?
7) Find $\lim_{n\to\infty} \sqrt[n]{n}$
8) Define Monotonic .
9) Write the formula for total variation.
10) If $f_n(x)=n^2x(1-x^2)^n$, then find $\int_0^1 \left[\lim_{n\to\infty} fn\ (x)\right] dx.$

**SECTION – B(5X3=15 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**

11. (a) Define Bolzano's statement.

**OR**

(b) Let f: S→T is a function from S to J. If x≤ $s$ and y≤ $T$ , then prove that $x=f^{-1}(y)$ implies f(x)≤ $y$

12.(a) Write the definition of derivatives.

**OR**

(c) Define zero derivatives.

13. (a) State mean- value theorem .

**OR**

(b) State generalized mean value theorem.

14. (a) Define total variation.

**OR**

(b)Define the Riemann-Stieljes integral.

15) (a) Define step function.

**OR**

(b) State Euler's summation formula.

**SECTION-C (5X5=25 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 & K5)**

16) (a) Show that a metric space S is the connected if and only if every two-valued function on S is constant.

**OR**

(b) If f is increasing on [a,b], then prove that f $(c^+)$ and f$(c^-)$ both exist for each c in (a,b) and we have f $(a) \leq f(a^+)$ and f $(b^-) \leq$f (b)

17) (a) State and prove chain rule theorem.

**OR**

(b) If f is differentiable at c, then prove that f is continuous at c.

18) (a) State and prove Taylor theorem.

**OR**

(b)If f is monotonic on [a, b], then prove that the set of discontinuities of f is countable.

19) (a) If f is monotonic on [a,b],then prove that f is bounded variation on [a,b].

**OR**

(b) If f$\in R(\propto)$ and if g$\in r(\propto)$ on [a, b], then prove that $c_1f + c_2g \in R(\propto)$ on [a, b] (for any two constant $c_1$ and $c_2$) and we have

$\int_a^b (c_1 f + c_2 g) d\propto = c_1 \int_a^b f\, d\propto + c_2 \int_a^b g\, d\propto$

20) (a) If f$\in R(\propto)$ $on\ [a,b], then\ prove\ that\ \propto \in R\ (f) on\ [a,b] and\ we\ have$

$$\int_a^b f(x) d \propto (x) + \int_a^b \propto (x) d\, f(x) = f(b) \propto (b) 0 - f(a) \propto (a).$$

**OR**

(b) Assume that $\propto\uparrow on$ [a, b], then prove that

i) $P^1$ finer than P have $\cup (p^1, f, \propto) \leq \cup (p, f, \propto)$and $L(p^1, f, \propto) L(p, f, \propto)$.

ii) For any two partitions $P_1$ and $P_2$, we have $L(P_1, f, \propto) \leq \cup (P_2, f, \propto)$

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