

(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22UMS6E6

REG.NO.:

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY-2025

B.Sc.-MATHEMATICS

MAXIMUM MARKS: 50

SEMESTER: VI

TIME: 3 HOURS

PART - III DISCRETE MATHEMATICS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS.

(K1)

- 1. The value of S(2) = _____, when the recurrence relation on S is S(k) - 7S(k - 1) + 12S(k - 2) = 0; S(0) = S(1) = 4. a. 20 b. -20 c. 76 d. -76
2. The dual of P uparrow (Q anddownarrow (R downarrow P) and T) = _____. a. P downarrow (Q vdownarrow (R uparrow P) v F) b. P downarrow (Q vdownarrow (R uparrow P) v T) c. P uparrow (Q vdownarrow (R downarrow P) v F) d. P uparrow (Q vdownarrow (R downarrow P) v T)
3. In the poset (D(12), <=), where <= is the relation 'is a divisor', 4 v 6 = _____. a. 3 b. 6 c. 12 d. 2
4. The maximum number of edges in a complete bipartite graph with n vertices are _____. a. n/2 b. n^2/2 c. n/4 d. n^2/4
5. If Sigma = {0,1,2}, then w = 01212 then |w| = _____. a. 5 b. 6 c. 3 d. 0

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.

(K2)

- 6. Write the recursive definition of a polynomial.
7. Prove that (p and (p <-> q)) -> q is a tautology.
8. Show that the lattice D(18) is not a Boolean algebra.
9. Can you make a simple undirected graph of eight vertices have 40 edges excluding self-loops?
10. Construct a finite-state machine which will accept all words over the input set Sigma = {a, b}, where number of b's are divisible by 3.

SECTION - B

(5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K3)

- 11. a) Find the recurrence relation satisfying y_n = (A + Bn)4^n. (OR) b) Solve S(k) - S(k - 1) - 6S(k - 2) = -30 where S(0) = 20, S(1) = -5.
12. a) Show that (not P and (not Q and R)) v (Q and R) v (P and R) <-> R. (OR) b) Obtain a conjunctive normal form of not (P v Q) <-> (P and Q).

13. a) Prove that every chain is a lattice.
(OR)
b) If $D(n)$ denotes the lattice of all positive divisors of the integer n , draw the Hasse diagrams of $D(10)$, $D(15)$, $D(32)$ and $D(45)$.
14. a) Prove that the number of vertices of odd degree in a graph is always even.
(OR)
b) If a simple graph G with n vertices has more than $\frac{1}{2}(n-1)(n-2)$ edges, then prove that G is connected.
15. a) Find the phase-structure grammar that generate the set $L = \{a^n b^{2n} : n \geq 1\}$.

(OR)

- b) Consider the finite automaton whose transition function
- f
- is given by

| Next-state Table | | |
|------------------|------------|-------|
| State | f (Inputs) | |
| | 0 | 1 |
| s_0 | s_2 | s_1 |
| s_1 | s_3 | s_0 |
| s_2 | s_0 | s_3 |
| s_3 | s_2 | s_2 |

where $S = \{s_0, s_1, s_2, s_3\}$, $\Sigma = \{0,1\}$, $F = \{s_0\}$. Write the entire sequence of states for the input string 110101.

SECTION – C**(5 X 5 = 25 MARKS)****ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.****(K4 (Or) K5)**

16. a) Write the recurrence relation for Fibonacci numbers and solve it.
(OR)
b) Find the generating function of the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0$ given $y_1 = 1, y_0 = 2$.
17. a) Show that $(P \rightarrow (Q \rightarrow R)) \Rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$.
(OR)
b) Obtain the principal disjunctive normal form of $P \rightarrow ((P \rightarrow Q) \wedge \neg (\neg Q \vee \neg P))$.
18. a) Prove that $(L \times M, \wedge, \vee)$ is a Lattice
(OR)
b) Let L be a distributive lattice and $a, b, c \in L$. If $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$, then prove that $b = c$.
19. a) Explain the types of Graphs.
(OR)
b) State and prove Euler theorem.
20. a) Explain the types of Grammar.
(OR)
b) Construct *dfa* equivalent to *nfa* whose state table is shown below:

| State | f (Inputs) | |
|-------|----------------|----------------|
| | a | b |
| s_0 | $\{s_0, s_1\}$ | $\{s_2\}$ |
| s_1 | $\{s_0\}$ | $\{s_1\}$ |
| s_2 | ϕ | $\{s_0, s_1\}$ |
