(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22UMS6E4

REG.NO.:

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY-2025

B.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: VI **TIME: 3 HOURS**

PART - III
THEORY OF NUMBERS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. If m and n are positive integers and if $m > 1$, then _______.

   a.$n > m$ b.$n = m$ c.$n < m$ d.$n \neq m$

2. If p is a prime, then ______________.

   a. $p|[(p-1)!+1]$ b. $p|[(p-1)!-1]$
   c. $p|[(p+1)!+1]$ d. $p|[(p+1)!-1]$

3. Since g.c.d.(15,12) = 3 and $3|9$, then the congruence $15x \equiv 9(mod\ 12)$ has exactly ______ mutually incongruent solutions.

   a.9 b.3 c.12 d.15

4. $d(120) =$ ______.

   a. 40 b. 30 c. 360 d. 16

5. $\lim_{x\to\infty} \pi(x)$is _____.

   a. $\neq \infty$ b. ∞ c. = 1 d. = 0

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define linear Diophantine equation.
7. State Cancellation law.
8. StateFermat's Little theorem.
9. Define $\mu(n)$.
10. Define a primitive root.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K3)

11. a) If x is any real number other than 1, then prove that $\sum_{j=0}^{n-1} x^j = \frac{x^n-1}{x-1}$.

(OR)

b) If a, b and c are integers, where a and c are relatively prime and if $c|ab$, then prove that c divides b.

(CONTD …. 2)

12. a) Prove that the of any n consecutive positive integers is divisible by the product of the first n positive integers.

(OR)

b) If s integers $r_1, r_2, \ldots\ldots r_s$ form a reduced residue system modulo m, then prove that $s = \phi(m)$.

13. a) State and prove Euler's theorem.

(OR)

b)Prove that if p is prime congruent to 1 modulo 4, then $\left\{\left(\frac{p-1}{2}\right)!\right\}^2 \equiv -1(mod\ p)$.

14. a) Prove that $\sum_{d|n} \phi(n) = n$.

(OR)

b) Prove that if one of the functions in the Mobius pair $\{f(n), g(n)\}$ is multiplicative, so is the other.

15. a) If g is a primitive root modulo m, then prove that $g, g^2, \ldots\ldots g^{\phi(m)}$ are mutually incongruent and form a reduced residue system modulo m.

(OR)

b) If p is a prime, then show that $\sum_{j=1}^{\infty}\left[\frac{n}{p^j}\right]$ is the exponent of p appearing in the prime factorization of $n!$.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a). State and prove Euclid's division lemma.

(OR)

b). State and prove fundamental theorem of arithmetic.

17. a) Prove that $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$.

(OR)

b) Prove that $a_n = \binom{2n-2}{n-1}/n$.

18. a) Prove that the congruence $(m-1)! \equiv -1(mod\ m)$ holds if and only if m is a prime.

(OR)

b) State and prove Chinese remainder theorem.

19. a) If $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots\ldots\ldots p_s^{\alpha_s}$, then prove that $d(n) = (\alpha_1+1)(\alpha_2+1)\ldots\ldots\ldots(\alpha_s+1)$, and $\sigma(n) = \frac{p_1^{(\alpha_1+1)}-1}{p_1-1}\frac{p_2^{(\alpha_2+1)}-1}{p_2-1}\ldots\ldots\ldots\frac{p_s^{(\alpha_s+1)}-1}{p_s-1}$.

(OR)

b) State and prove Mobius inversion frmula.

20. a) Prove that for each prime p, there exist primitive roots modulo p.

(OR)

b) Find the value of $\lim_{x\to\infty} \frac{\pi(x)}{x}$.

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