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(FOR THE CANDIDATES ADMITTED **SUBJECT CODE** **24 PPS 101**

DURING THE ACADEMIC YEARS 2024-26 ONLY) **REG.NO**

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER – 2024

M.Sc. – PHYSICS **MAXIMUM MARKS: 50**

I SEMESTER **TIME : 3 HOURS**

PART – III

MATHEMATICAL PHYSICS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. **(K1)**

1. The value of $P_n(1)$ is

   a) 0 b) 1 c) -1 d) $(-1)^n$

2. Which of the following is not an analytic function?

   a) $|z|$ b) sin z c) z^{-1} d) none of these

3. Heat flow equation is

   a) $\nabla^2 \Phi = 1$ b) $\nabla^2 \Phi = 0$ c) $\nabla^2 \Phi = \frac{1}{h^2} \frac{\partial \Phi}{\partial t}$ d) $\nabla^2 \Phi = \rho$

4. The Fourier transform of the Gaussian function $f(x) = e^{-x^2}$ is

   a) $\frac{1}{\sqrt{2}} e^{-\omega^2/4}$ b) $\frac{1}{\sqrt{2\pi}} e^{-\omega^2/2}$ c) $\frac{1}{\sqrt{2\pi}} e^{-\omega^2}$ d) None of these

5. The value of $\Gamma(0)$ is

   a) 0 b) 1 c) -1 d) ∞

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. What is the value of the Bessel function $J_{3/2}(x)$?
7. Write the Cauchy Riemann equations in polar form.
8. Write the Laplace's equation in cylindrical coordinates.
9. Write the complex form of Fourier integral for $f(x)$
10. What is a dummy index?

(CONTD …. 2)

SECTION – B (5 X 5 = 25)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. (a) Obtain the generating function of Hermite polynomials

(OR)

(b) Prove the recurrence relation $n\,P_n = (2n-1)x\,P_{n-1} - (n-1)\,P_{n-1}$

12. (a) State and prove Cauchy's integral theorem.

(OR)

b) Determine the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.

13. (a) Obtain the solution of Laplace's equation in Cartesian coordinates.

(OR)

(b) Find the solution of heat flow equation by the method of separation of variables.

14. (a) Express the function $f(x) = \begin{cases} 1 & for\ |x| \le 1 \\ 1 & for\ |x| > 1 \end{cases}$ as a Fourier integral and hence evaluate $\int_0^\infty \frac{\sin\lambda\, cos\lambda x}{\lambda}\, d\lambda$

(OR)

(b) State and prove Parseval's theorem of Fourier transform.

15. (a) Define Kronecker delta symbol and mention its properties.

(OR)

(b) Derive the relation between Beta and Gamma functions.

SECTION – C (5 X 8 = 40)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4/K5)

16. a) Solve the Bessel's differential equation and obtain its most general solution.

(OR)

b) Prove the orthogonal properties of Legendre polynomials.

(CONTD …. 3)

17. a) Find the Laurent's series expansion of $f(z) = \frac{1}{(z+1)(z+3)}$ valid for

(i) $|z| < 1$

(ii) $1 < |z| < 3$

(OR)

b) Evaluate $\int_0^{2\pi} \frac{4}{5+4\cos\theta} d\theta$ using calculus of residues.

18. a) Obtain the solution of Laplace's equation in spherical polar coordinates.

(OR)

b) Determine the steady state temperature distribution of a thin plate bounded by the lines $x = 0, x = l, y = 0, y = \infty$ assuming that heat cannot escape from either surface of the plate, the edges $x = 0, x = l, y = \infty$ are maintained at zero temperature and the edge $y = 0$ is maintained at steady state temperature $F(x)$.

19 .a) Find Fourier transform of the function

$$f(x) = 1 - x^2; \quad -1 < x < 1$$
$$= 0 \quad ; \quad |x| = 1$$

and hence evaluate $\int_0^{\infty} \left(\frac{x\,cosx - sinx}{x^3} \right) cos\frac{x}{2}\, dx$

(OR)

b) Find the Fourier sine and cosine transforms of $f(t) = e^{-pt}$, $p > 0$. Hence evaluate the integrals (i) $\int_0^{\infty} \frac{\cos\omega t}{p^2+\omega^2} d\omega$ (ii) $\int_0^{\infty} \frac{\omega \sin\omega t}{p^2+\omega^2} d\omega$

20. a) Show that in Cartesian coordinate system the contravariant and covariant components of a vector are identical.

(OR)

b) Show that $\int_0^{\pi/2} sin^p\theta\, cos^q\theta\; d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$

Hence evaluate $\int_0^{\pi/2} sin^p\theta\; d\theta$ and $\int_0^{\pi/2} cos^p\theta\; d\theta$

ETHICAL PAPER

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