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(FOR THE CANDIDATES ADMITTED 23UDA3A1

DURING THE ACADEMIC YEAR _ 2024 ONLY) REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER 2024

B.Sc (Computer Science with Data Analytics) **MAXIMUM MARKS: 75**

SEMESTER:III **TIME : 3 HOURS**

PART - III

23UDA3A1 - INTRODUCTION TO LINEAR ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

1. If A is an m X n matrix and m < n, then the homogeneous system of linear equation has a __

(a) non-trivial solution (b) unique solution (c) exact solution (d) trivial solution

2. If A and B are matrices of the same order, A + B = B + A is said to be.________

(a) Commutative (b) Associative (c) Distributive (d) Identity

3. If any two rows or columns of a determinant are identical, the value of the determinant is ___

(a) Zero (b) One (c) Empty (d) constant

4. The system is defined by the state matrix A = $\begin{bmatrix} -2 & -4 \\ 1 & 0 \end{bmatrix}$. The system is__________

(a) Undamped (b) Underdamped (c) Critically damped (d) Overdamped

5. What is the dimension of R^n matrix?

(a) n + 1 (b) n (c) n – 1 (d) 1

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES

6. What is meant by row echelon?

7. Define skew matrix.

8. What is the meant by inverse matrix?

9. Define the characteristic equation of a matrix.

10. What is the standard ordered Bases for P_3 (R)?

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SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

11. a) Prove that every superset of a linearly dependent set is linearly dependent.

(OR)

b) Solve the matrix equation Ax = b if A = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 6 \end{bmatrix}$, **b** = $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

12. a) If A= $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$, B= $\begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, find AB and BA and show that AB≠BA.

(OR)

b) Show that the value of the determinant of skew-symmetric matrix of odd order is always zero.

13. a) Discuss the cofactors of the elements of the determinant for the following problem

$\Delta= \begin{vmatrix} 2 & 3 & 2 \\ 1 & 4 & -1 \\ 5 & 6 & 8 \end{vmatrix}$

(OR)

b) Find the adjoint of the matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

14. a) Explain matrix polynomial with an example.

(OR)

b) Determine the eigen values of the given matrix: A= $\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & c & c \end{bmatrix}$

15. a) Show that the transformation T : V_2 (R) $\rightarrow$ V_2 (R) defined by T (a, b) = (a + b, a) $\forall$ a, b $\in$ R is a linear transformation.

(OR)

b) Show that the set V = { (x, y) $\in$ R^2 | xy $\geq$ 0 } is not a vector space of R^2.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

16. a) Solve the following system of linear equations with the help of Cramer's rule: x+2y+3z=6, 2x+4y+z=7, 2x+2y+9z=14.

(OR)

b) Show that the vectors [2 3 -1 -1],[1 -1 -2 -4], [3 1 3 -2], [6 3 0 -7] form a linearly dependent set. Also express one of these as a linear combination of the others.

17. a) If A and B are n-rowed square matrices, show that

i) $(A+B)^2=A^2+AB+BA+B^2$

ii) $(A+B)(A-B)=A^2-AB+BA-B^2$

iii) $(A-B)^2=A^2-AB-BA+B^2$

(OR)

b) Show the every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew-symmetric matrix.

18. a) Find the adjoint of a matrix $A = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 1 & 1 \\ 4 & -5 & 2 \end{bmatrix}$

(OR)

b) Find the rank of the matrix $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing to normal form.

19. a) Distinguish the relation between Algebraic and Geometric multiplicities of a characteristic root.

(OR)

b) Determine the characteristic roots and corresponding characteristic vectors of the given matrix. $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

20. a) Show that the given subset of vectors of R^3 forms a basis for R^3 {(1,2,1), (2,1,0), (1, -1,2)}

(OR)

b) Given a linear transformation T on $V_3(R)$ defined by T(a, b, c) = (2b + c, a – 4b, 3a) corresponding to the basis B = {(1, 0, 0), (0, 1, 0), (0,0,1)}.

Find the matrix representation of T.

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