(FOR THE CANDIDATES ADMITTED
 DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS314

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024
COURSE NAME: M.Sc. MATHEMATICS   MAXIMUM MARKS: 75
SEMESTER: III   TIME : 3 HOURS

GRAPH THEORY

SECTION – A   **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS.   **(K1)**

1. The number of edges in $K_{4,5}$ is _____.
      a) 9   b) 20   c) 1   d) 5

2. A connected graph has a Euler trail if and only if it has _____ vertices of odd degree.
      a) At most 2   b) At least 2   c) Exactly 2   d) Exactly 1

3. For a graph with 12 vertices, if $\alpha = 3$ then $\beta =$ _____.
      a) 7   b) 20   c) 9   d) 5

4. If G is k-critical then _____.
      a) $\delta \leq 4$ b) $\delta = 4$ c) $\delta = 5$ d) $\delta \geq 4$

5. If G is a simple planar graph with $v \geq 3$, then _____.
      a) $\varepsilon \leq 3v$-6   b) $\varepsilon = 3v$-6   c) $\varepsilon \leq 3v$-5   d) $\varepsilon = 3v$-6

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.   **(K2)**
6. Define cut edge of a graph.
7. Define block of a graph.
8. Define M-alternating path.
9. Define chromatic number of a graph with example.
10. Explain disconnected graph.

SECTION – B   **(5 X 5 = 25 MARKS)**
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.   **(K3)**

11. a)   Prove that $\sum_{v \in V} d(v) = 2\varepsilon$

(OR)
  b) Prove that in a tree, any two vertices are connected by a unique path.

12. a) If G is a block with $v \geq 3$, then any two edges of G lie on a common cycle.
(OR)
  b) If G is a simple graph with $v \geq 3$ and $\delta \geq v/2$, then show that G is Hamiltonian.

13. a)   Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.
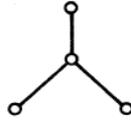**(OR)**
  b) Prove that a set S V is an independent set of G if and only if V\S is a covering of G.

14. a)   In a critical graph, prove that no vertex cut is a clique.

(OR)

   b)   Find the chromatic polynomial for the graph.

15. a)   If G is a connected plane graph, then prove that $v - \varepsilon + \phi = 2$.

(OR)

   b)   Prove that a loopless digraph D has an independent set S such that each vertex of D not in S is reachable from a vertex in S by a directed path of length at most two.

SECTION – C   **(5 X 8 = 40 MARKS)**
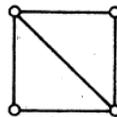**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**
(K4 (Or) K5)

16. a)   Prove that a graph is bipartite if and only if it contains no odd cycle.

(OR)

   b)   Find the number of spanning trees $\tau(G)$ for the graph.

17. a)   Prove that a graph G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

(OR)

   b)   State and Prove Chvatal's Theorem.

18. a)   Prove that G has a perfect matching if and only if $O(G - S) \leq |S|$ for all $S \subset V$.

(OR)

   b)   Prove that for $\delta > 0$, $\alpha' + \beta' = v$.

19. a)   If G is simple, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

(OR)

   b)   If G is a connected simple graph and is neither an odd cycle nor a complete graph, then show that $\chi \leq \Delta$.

20. a)   Prove that a graph is planar if and only if it contains no subdivision of K_5 or $K_{3,3}$.

(OR)

   b)   Prove that a digraph D contains a directed path of length $\chi - 1$.

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