

(FOR THE CANDIDATES ADMITTED  
DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS311

REG.NO. :

**N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI**  
**END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024**  
**COURSE NAME: M.Sc.- MATHEMATICS** **MAXIMUM MARKS: 75**  
**SEMESTER: III** **TIME : 3 HOURS**

### TOPOLOGY

#### SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS.

(K1)

1. A subset A of a topological space X is said to be closed if the set \_\_\_\_\_ is open.  
a) X-A                      b) X                      c) A                      d) X+A
2. The standard metric on the real numbers R is defined by the equation on  $d(x, y) = \underline{\hspace{2cm}}$ .  
a)  $|x + y|$                       b)  $|x - y|$                       c)  $|x|$                       d)  $|y|$
3. A space X is said to be \_\_\_\_\_ if every open covering A of X contains a finite subcollection that also covers X.  
a) compact                      b) bounded                      c) Not compact                      d) none
4. A subset A of a space X is said to be dense in X, if \_\_\_\_\_.  
a)  $A = X$                       b)  $A = -X$                       c)  $\bar{A} = X$                       d)  $\bar{A} = -X$
5. Two compactifications  $Y_1$  and  $Y_2$  of X are said to be equivalent, if there is a homeomorphism  $h: Y_1 \rightarrow Y_2$  such that \_\_\_\_\_ for every  $x \in X$ .  
a)  $h(x) = x$                       b)  $h(x) = -x$                       c)  $h(x) = x^2$                       d)  $h(x) = -x^2$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.

(K2)

6. Define product topology.
7. Define metric topology.
8. State intermediate value theorem.
9. State Urysohn metrization theorem.
10. Define paracompact space.

#### SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Let Y be a subspace of X. If U is open in Y and Y is open in X, then prove that U is open in X.

(OR)

- b) Prove that every finite point set in a Hausdorff space X is closed.

12. a) State and prove Pasting Lemma.

(OR)

- b) Let  $f: X \rightarrow Y$ . If the function f is continuous, then show that for every convergent sequence  $x_n \rightarrow x$  in X, the sequence  $f(x_n)$  converges to  $f(x)$ .

13. a) Prove that the image of a connected space under a continuous map is connected.

(OR)

- b) Prove that every closed subspace of a compact space is compact. (CONTD....2)

14. a) Suppose that  $X$  has a countable basis. Show that every open covering of  $X$  contains a countable subcollection covering  $X$ .

(OR)

b) State Tietze extension theorem.

15. a) Prove that every paracompact Hausdorff space  $X$  is normal.

(OR)

b) Let  $A \subset X$ ; let  $f: A \rightarrow Z$  be a continuous map of  $A$  into the Hausdorff space  $Z$ . Prove that there is at most one extension of  $f$  to a continuous function  $g: \bar{A} \rightarrow Z$ .

**SECTION – C**

**(5 X 8 = 40 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)**

16. a) Prove that the topologies of  $R_l$  and  $R_k$  are strictly finer than standard topology in  $R$ , but are not comparable with one another. (K4)

(OR)

b) Let  $Y$  be a subspace of  $X$ . Prove that a set  $A$  is closed in  $Y$  if and only if it equals the intersection of a closed set of  $X$  with  $Y$ . (K4)

17. a) Let  $X$  be a metric space with metric  $d$ . Define  $\bar{d}: X \times X \rightarrow R$  by the equation  $\bar{d}(x, y) = \min\{d(x, y), 1\}$ . Prove that  $\bar{d}$  is a metric that indicates the same topology as  $d$ . (K5)

(OR)

b) State and prove Uniform limit theorem. (K5)

18. a) Prove that a finite cartesian product of connected spaces is connected. (K4)

(OR)

Prove that a space  $X$  is locally connected if and only if for every open set  $U$  of  $X$ , each component of  $U$  is open in  $X$ . (K4)

19. a) Prove that every regular space with a countable basis is normal. (K4)

(OR)

b) State and prove Urysohn Lemma. (K4)

20. a) State and prove Tychonoff Theorem. (K5)

(OR)

b) Prove that a space  $X$  is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable. (K5)

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