(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS103

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024
COURSE NAME: M.Sc.- MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: I **TIME : 3 HOURS**

COMPLEX ANALYSIS

SECTION – A (10 X 1 = 10 MARKS)
ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. If f(z) is analytic in a region Ω, then $\frac{1}{2\pi i}\int_{\gamma} \frac{f(z)\,dz}{z-a} =$_______, for every cycle γ which is homologous to zero in Ω.

   a) n(γ, a) b) n(γ, a)f(a) c) f(a) d) None

2. A function u(x,y) is harmonic, if it satisfies__________

   a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ b) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$ c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$ d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -1$

3. The value of $\Gamma\left(\frac{1}{2}\right) =$ _______.

   a) π b) $-\pi$ c) $\sqrt{\pi}$ d) $-\sqrt{\pi}$

4. $\frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-z_0)^2} =$ ______

   a) f(z) b) $f'(z)$ c) $f''(z)$ d) None

5. The sum of the residues of an elliptic function is _______

   a) zero b) one c) two d)three

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define simply connected region.
7. State the maximum principle for harmonic functions.
8. State Hurwitz Theorem.
9. Define Equicontinuous.
10. Write the complex Fourier development of f(z).

SECTION – B (5 X 5 = 25 MARKS)
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) State and prove Argument Principle.

(OR)

   b) Find the pole and residue of f(z) $= \frac{1}{z^2+5z+6}$.

12.a) State the Mean - Value Theorem.

(OR)

   b) State the Reflection Principle.

13.a) State and prove Weirstrass's Theorem.

(OR)

   b) Show that $\frac{\pi}{\sin \pi z} = \lim_{m\to\infty} \sum_{-m}^{m} (-1)^n \frac{1}{z-n}$.

(CONTD…..2)

14. a) State Arzela's Theorem.

(OR)

b) State Poisson -Jensen Formula.

15. a) Prove that an elliptic function without poles is a constant .

(OR)

b) Write a note on Simple Periodic Functions.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) State and Prove Cauchy's Theorem. **(K5)**

(OR)

b) Evaluate $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$, $a > 1$ **(K5)**

17.a) Derive Poisson's Formula. **(K4)**

(OR)

b) State and prove Schwarz's theorem. **(K4)**

18. a) State and prove Taylor's Series. **(K4)**

(OR)

b) Prove that the infinite product $\prod_1^{\infty}(1+a_n)$ with $1+a_n \neq 0$ converges simultaneously with the series $\sum_1^{\infty}\log(1+a_n)$ whose terms represent the values of the principal branch of the logarithm. **(K4)**

19.a) Derive Jensen's formula. **(K4)**

(OR)

b) Prove that a family Fof analytic functions is normal with respect to C if and only if the functions in F are uniformly bounded on every compact set. **(K4)**

20.a) Prove that the zeros $a_1, a_2, a_3, \ldots\ldots, a_n$ and poles $b_1, b_2, b_3, \ldots\ldots, b_n$ of an elliptic function satisfy $a_1 + a_2 + a_3 + \cdots\ldots + a_n = b_1 + b_2 + b_3 + \ldots\ldots + b_n (mod\ M)$. **(K5)**

(OR)

b) Explain Unimodular transformations **(K5)**

****