

(FOR THE CANDIDATES ADMITTED  
DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS102

REG.NO. :

**N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI**  
**END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2024**  
**COURSE NAME: M.Sc.-MATHEMATICS** **MAXIMUM MARKS: 75**  
**SEMESTER: I** **TIME: 3 HOURS**

**REAL ANALYSIS**

**SECTION – A (10 X 1 = 10 MARKS)**

**ANSWER THE FOLLOWING QUESTIONS.**

**MULTIPLE CHOICE QUESTIONS. (K1)**

- $f \in R(\alpha)$  on  $[a, b]$  iff there exists a partition  $P$  of  $[a, b]$  such that  $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$   
(a)  $<$  (b)  $>$  (c)  $=$  (d)  $\neq$
- Every uniformly convergent sequence is \_\_\_\_\_.  
(a) pointwise convergent (b) divergent (c) not convergent (d) not divergent
- If  $\phi$  maps a metric space  $X$  into  $X$  and if there is a number  $c < 1$  such that  $d(\phi(x), \phi(y)) \leq cd(x, y) \forall x, y \in X$ , then  $\phi$  is said to be \_\_\_\_\_.  
(a) continuous (b) differentiable (c) contraction (d) deletion
- The outer measure of a countable set is \_\_\_\_\_.  
(a) finite (b) infinite (c) zero (d) one
- A non - negative measurable function  $f$  is called integral over the measurable set  $E$ , if  $\int_E f < \infty$ .  
(a) 0 (b)  $\infty$  (c) 1 (d) -1

**ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)**

- Define a closed curve.
- Define equicontinuous..
- Define vector space
- Give the canonical representation of the simple function  $\phi$ .
- Define the Lebesgue integral.

**SECTION – B (5 X 5 = 25 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**

- a) Prove that if  $f$  is monotonic and  $\alpha$  is continuous on  $[a,b]$ , then  $f \in R(\alpha)$  on  $[a,b]$ .  
(OR)  
b) State and prove theorem on Integration by parts .
- a) State and prove the Cauchy Criterion for uniform convergence of sequence of functions.  
(OR)  
b) If  $K$  is a compact metric space and if  $f_n \in C(K)$  for  $n = 1, 2, \dots$  and if  $\{f_n\}$  converges uniformly on  $K$ , then prove that  $\{f_n\}$  is equicontinuous on  $K$ .

13.a) If  $\Omega$  be the set of all invertible linear operators on  $\mathbb{R}^n$ . If  $A \in \Omega$  and  $B \in L(\mathbb{R}^n)$  such that  $\|B - A\| \|A^{-1}\| < 1$ , then prove that  $B \in \Omega$ .

(OR)

b) State and prove Fixed Point theorem.

14.a) If  $m^* E = 0$ , then prove that E is measurable.

(OR)

b) Let c be a constant and f and g are two measurable real valued functions defined on the same domain, then show that f+c, cf, f+g, f-g and fg are measurable.

15.a) State and prove Fatou's Lemma.

(OR)

b) State and prove Monotone convergence theorem.

**SECTION – C****(5 X 8 = 40 MARKS)****ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.****(K4 (Or) K5)**

16. a) If  $\gamma'$  is continuous on  $[a, b]$ , then prove that  $\gamma$  is rectifiable and

$$L(\gamma) = \int_a^b |\gamma'(t)| dt.$$

(OR)

b) Suppose 'f' is bounded on  $[a, b]$ , f has only finitely many discontinuity on  $[a, b]$  and  $\alpha$  is continuous at every point at which f is discontinuous. Show that  $f \in R(\alpha)$  on  $[a, b]$ .

17.a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

(OR)

b) State and prove Stone Weierstrass theorem

18. a) Prove the following.

(i) If  $A \in L(\mathbb{R}^n, \mathbb{R}^m)$ , then  $\|A\| < \infty$  and A is a uniformly continuous mapping of  $\mathbb{R}^n$  to  $\mathbb{R}^m$ .

(ii) If  $A, B \in L(\mathbb{R}^n, \mathbb{R}^m)$  and c is a scalar, then

$$\|A + B\| \leq \|A\| + \|B\|, \|cA\| = |c| \|A\|.$$

(iii) If  $A \in L(\mathbb{R}^n, \mathbb{R}^m)$  and  $B \in L(\mathbb{R}^m, \mathbb{R}^k)$ , then  $\|BA\| \leq \|B\| \|A\|$ .

(OR)

b) State and prove Implicit function theorem.

19.a) Let  $\{E_n\}$  be an infinite decreasing sequence of measurable sets, that is a sequence with  $E_{n+1} \subset E_n$  for each n. Let  $mE_1$  be finite. Then prove that

$$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \rightarrow \infty} mE_n$$

(OR)

b) Prove that the outer measure of an interval is its length..

20.a) State and prove Lebesgue convergence theorem.

(OR)

b) State and prove Bounded Convergence Theorem.