(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2024 ONLY)

24PMS101

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2024
COURSE NAME: M.Sc.-MATHEMATICS   MAXIMUM MARKS: 75
SEMESTER: I   TIME: 3 HOURS

ALGEBRA

SECTION – A   (10 X 1 = 10 MARKS)
ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS.   (K1)

1. If G is a group and if $a \in G$, then the set $N(a) = \{x \in G \,/\, xa = ax\}$ is called the _____.
   (a) Conjugacy   (b) normal subgroup   (c) Normalizer   (d) none of these

2. If R is a commutative ring, then $a \neq 0 \in R$ is said to be a _____ if there exists $a, b \in R, \; b \neq 0$, such that $ab = 0$.
   (a) zero-divisor   (b) integral domain   (c) Division ring   (d) field

3. The element $a \in K$ is algebraic over F if and only if F(a) is a _____ of F
   (a) Degree   (b) finite extension   (c) Algebraic extension   (d) none of these

4. if $u, v \in V$ then u is said to be_____ to v if (u, v)=0
   (a) orthogonal   (b) inner product   (c) orthogonal complement (d) orthogonal set

5. Any finite extension set, then the list of characteristic 0 is a _____
   (a) simple extension   (b) algebraic extension   (c) finite extension (d) extension

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.   (K2)
6. Define conjugate of *a* in G.
7. State the Pigeonhole Principle.
8. Define Algebraic extention.
9. Define norm of *v*.
10. Define the fixed field of a group G.

SECTION – B   (5 X 5 = 25 MARKS)
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)
11. a) If o(G) = p^2 where p is a prime number , then G is abelian.
                              (OR)
   b) If $p^m \,|\, o(G), \; p^{m+1} \nmid o(G)$, then G has a subgroup of order p^m .

12. a)   State and Prove the Gauss Lemma.
                              (OR)
   b) If R is an integral domain, then prove that R[x] is also an integral domain.

13. a)   If F is an algebraic extension of K and if K is an algebraic extension of F , then prove that L is an algebraic extension of F .
                              (OR)
   b)   Let F be a finite field with q elements and suppose that $F \subset K$ where K is also a finite field. Then prove that K has q^n elements where $n = [K : F]$

14. a) Prove that W^T is a subspace of V

(OR)

b) If $p(x) \in F[x]$ and if K is an extension of F , then prove that for any element $b \in K$, $p(x) = (x - b)q(x) + p(b)$ $where$ $q(x) \in K[x]$ and where deg $q(x) = $ deg $p(x) - 1$

15. a) Prove that the fixed field of G is a subfield of K

(OR)

b) If K is a finite extension of F, then prove that G(K, F) is a finite group and its order, o(G(K,F)) satisfies o(G(K,F)) $\leq [K : F]$

SECTION – C   (5 X 8 = 40 MARKS)
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.
(K4 (Or) K5)

16. a) If $o(G) = p^2$ where p is a prime number, then $Z(G) \neq (e)$

(OR)

b) State and prove the third part of the Sylow's theorem

17. a) State and prove Division Algorithm

(OR)

b) State and prove The Eisenstein Criterion

18. a) If L is a finite extension of K and if K is a finite extension of F ,then prove that L is a finite extension of F .Moreover,$[L : F] = [L : K][K : F]$.

(OR)

b) Let G be a finite abelian group enjoying the property that the relation $x^2 = e$ is satisfied by at most n elements of G , for every integer n. Then prove that G is a cyclic group.

19. a) If $u, v \in V$ then prove that $|(u, v)| \leq \|u\| \|v\|$

(OR)

b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

20. a) If K is a field and if $\sigma_1 \cdots , \sigma_n$ are automorphisms of K, then prove that it is impossible to find elements $\sigma_1 \cdots , \sigma_n$, not all 0, in K such that $a_1 \sigma_1(u) + a_2 \sigma_2(u) + \cdots , + a_n \sigma_n(u) = 0$ for all $u \in K$

(OR)

b) State and prove Fundamental theorem of Galois theory

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