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(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2022 ONLY)

REG.NO.:

N.G.M. COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2024

COURSE NAME: MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: V **TIME: 3 HOURS**

PART - III

22UMS513-COMPLEX ANALYSIS

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
(Objective Questions with four Multiple Choices) **(K1)**

1. The fixed point of the linear transformation $z + a$ is ___________.

   a. 0 b. ∞ c. $-\infty$ d. 0 and ∞

2. The geometric series converges for __________.

   a. $|z| < 1$ b. $|z| \leq 1$ c. $|z| > 1$ d. $|z| \geq 1$

3. $\left|e^{iy}\right|^2$ is ________.

   a. < 1 b. > 1 c. $= 1$ d. ≥ 1

4. If $a \leqq b$, then $\left|\int_a^b f(t)dt\right|$ is ______

   a. $\geqq \int_a^b |f(t)|dt$ b. $> \int_a^b |f(t)|dt$ c. $< \int_a^b |f(t)|dt$ d. $\leqq \int_a^b |f(t)|dt$

5. If $\lim_{z \to a} f(z) = \infty$, then the point a is said to be a _________ of $f(z)$.

   a. removable singularity b. pole

   c. essential singularity d. zero

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**

6. Define continuous function.
7. Define uniform convergence of a function.
8. Write the C-R equations.
9. Find $\int_C \frac{dz}{z-a}$.
10. Define meromorphic function.

SECTION – B **(5 X 3 = 15 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.
(K3)

11. a) Prove that $u = x^2 - y^2$ is harmonic and find its analytic function.

(OR)

b) State and prove Lucas's theorem.

12. a) Show that the limit function of a uniformly convergent sequence of continuous functions is itself continuous.

(OR)

b) If $\sum_0^\infty a_n$ converges, then prove that $f(z) = \sum_0^\infty a_n z^n$ tends to $f(1)$ as z approaches 1 in such a way that $[1 - z]/(1 - |z|)$ remains bounded.

13. a) Explain briefly the Logarithm.

(OR)

b) Prove that the mapping $w = f(z)$ is said to be conformal at all points with $f'(z) \neq 0$.

14. a) Prove that the line integral $\int_\gamma p\, dx + q\, dy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$.

(OR)

b) If the piecewise differentials closed curve γ does not pass through the point a, then prove that the value of the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of $2\pi i$.

15. a) State and prove the Weierstrass theorem.

(OR)

b) State and prove the lemma of Schwarz.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Derive the C-R equations.

(OR)

b) Prove that every rational function has a representation by partial fractions.

17. a) Show that a sequence is convergent if and only if it is a Cauchy sequence.

(OR)

b) For every power series $a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n + \cdots$ there exists a number R, $0 \leq R \leq \infty$, called the radius of convergence, with the following properties:

(i). The series converges absolutely for every z with $|z| < R$. If $0 \leq \rho \leq R$, the convergence is uniform for $|z| \leq \rho$.

(ii). If $|z| > R$, the terms of the series are unbounded, and the series is consequently divergent.

18. a) Explain briefly the periodicity of a function $f(z)$.

(OR)

b) Prove that an analytic function in a region Ω whose derivatives vanishes identically must reduce to a constant. The same is true, if either the real part, the imaginary part, the modulus, or the argument is constant.

19. a) State and prove the Cauchy's theorem.

(OR)

b) Derive the Cauchy's integral formula.

20. a) State and prove the Taylor's theorem.

(OR)

b) State and prove the local mapping theorem..

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