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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2022 ONLY)

22UMS510

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: V **TIME : 3 HOURS**

PART - III

MODERN ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. HK is a subgroup of G iff ______________
   (a) H=K (b) $H^{-1}K=e$ (c) HK=KH (d) $HK \neq KH$
2. A subgroup N of G is a normal subgroup of G iff the product of two right cosets of N in G is____________
   (a)Odd (b) even (c) left cosets (d) right cosets
3. The product of two even permutations is __________permutation.
   (a) Odd (b) even (c) 0 (d) ∞
4. If R is a unit element 1, then $\frac{R}{U}$ has a unit element______.
   (a) U (b) 1 (c) 1+U (d) 1-U
5. A Euclidean ring possesses a _____ring.
   (a) Not a unit (b) unit (c) maximal ideal (d) principal ideal

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define abelian group.
7. Define homomorphism.
8. Let G be a group of order 99 and suppose that H is a subgroup of G of order 11 . Then find $i(H)$
9. What is called division ring ?
10. Define relatively prime.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) State and prove Fermat theorem.

(OR)

b) State and prove Lagrange's theorem.

12. a) Prove that the subgroup of G is a normal subgroup of G $\Leftrightarrow$ every left coset of N in G is a right coset of N in G.

(OR)

b) If G is a group, N be a normal subgroup of G, then prove that $\frac{G}{N}$ is also a group.

13. a) Find the orbits and cycles of the following permutations

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 8 & 1 & 6 & 4 & 7 & 5 & 9 \end{pmatrix}$ (ii)) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$

(OR)

b) If $o(G)= p^n$ where p is a prime number, then Prove that $Z(G) \neq (e)$.

(CONTD….2)

14.a) If ϕ is a homomorphism of R into R' with kernel $I(\phi)$, then prove that
(i) $I(\phi)$ is a subgroup of R under addition
(ii) If $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$.

(OR)

b) Prove that a finite integral domain is a field.

15. a) Prove that let R be a Euclidean ring and let A be an ideal of R. then there exists an element $a_0 \varepsilon A$ such that A consists exactly of all a_0x as x ranges over R.

(OR)

b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then R is a field.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) If H and K are finite subgroup of G of orders 0(H) and 0(K) respectively then prove that

$$0(HK) = \frac{0(H)0(K)}{0(H \cap K)}$$

(OR)

b) If G is a group then prove the following,
(i) The identity element of G is unique
(ii) Every $a \in G$ has a unique inverse in G
(iii) For every $a \in G$, $(a^{-1})^{-1} = a$
(iv) For all $a, b \in G$, $(a.b)^{-1} = b^{-1}.a^{-1}$

17. a) State and prove fundamental theorem of homomorphism.

(OR)

b) State and prove Cauchy's theorem for abelian Groups.

18. a) State and prove Cayley's theorem.

(OR)

b) State and prove Cauchy's theorem.

19.a) If R is a ring then prove the following for all $a, b \in R$
(i) a0=0a=0
(ii) a(-b)=(-a)b=(-ab)
(iii) (-a)(-b)=ab
If in addition, R has a unit element 1, then prove
(iv) (-1)a=-a
(v) (-1)(-1)=1

(OR)

b) If U is an ideal of the ring R, and then prove that $\frac{R}{U}$ is a ring and is a homomorphic image of R.

20.a) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a Maximal ideal of R iff $\frac{R}{M}$ is afield.

(OR)

b) Prove that J[i] is a Euclidean ring.

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