N.G.M COLLEGE (AUTONOMOUS): POLLACHI
END – OF - SEMESTER EXAMINATIONS: NOVEMBER 2024
COURSE NAME: B.Sc.-PHYSICS   MAXIMUM MARKS: 75
SEMESTER: I   TIME: 3 HOURS

PART-III
ANCILLARY MATHEMATICS FOR PHYSICS - I

SECTION – A
ANSWER THE FOLLOWING QUESTIONS: **(10 X1 =10 MARKS)**
MULTIPLE CHOICE QUESTIONS.
1. A real matrix is unitary iff it is _____.   **[K1]**
        a) scalar   b) not diagonal   c) skew symmetry   d) orthogonal
2. Every polynomial equation of nth degree has n roots and only ------- roots.
        a) 1   b) 2   c) n   d) zero

3. 'e' is -------- number.
        a) rational   b) irrational   c) infinity   d) none
4. In Gaussian elimination method, original equations are transformed by using --------.
        a) column operations   b) row operations c) mathematical operations d) subset operations
5. The value of $\beta(m,n)$ =--------.
        a) $\beta(1,n)$ b) $\beta(m,1)$ c) $-\beta(m,n)$ d) $\beta(n,m)$

ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES.   **[K2]**
6. Define unitary matrix.
7. Write the Conjugate pair of $\alpha + i\beta$.
8. Write the series of log 2.
9. Gauss elimination and Gauss Jordan elimination methods are?
10. Write the formula for $\Gamma\left(\dfrac{1}{2}\right)$.

SECTION – B   (5 X 5 = 25 MARKS)
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.   **[K3]**

11. a) Express $\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrices.

[OR]

b) Show that $\begin{bmatrix} \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{6}} & \dfrac{-1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{3}} & \dfrac{-2}{\sqrt{6}} & 0 \\ \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$ is orthogonal.

12. a) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in A. P.

[OR]

b) Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$ given that $2 + i\sqrt{3}$ is a root.   **(CONTD......2)**

13. a) Find the sum to infinity of the series $1 + \dfrac{2}{6} + \dfrac{2.5}{6.12} + \dfrac{2.5.8}{6.12.18} + \dots \infty$.

[OR]

b) Show that $2\left[1 + \dfrac{(\log n)^2}{2!} + \dfrac{(\log n)^4}{4!}\right] + \dots + \infty = n + \dfrac{1}{n}$.

14. a) Solve the system of equations using Gauss elimination method:

$x + 2y + z = 3$; $2x + 3y + 3z = 10$; $3x - y + 2z = 13$.

(OR)

b) Solve by Gauss Jordan method: $2x + 3y - z = 5$; $4x + 4y - 3z = 3$; $2x - 3y + 2z = 2$.

15. a) Derive the duplication formula $\Gamma m \, \Gamma\left(m + \dfrac{1}{2}\right) = \dfrac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m$. Hence show that

$$\beta(m, m) = 2^{1-2m} \, \beta\left(m, \dfrac{1}{2}\right).$$

[OR]

b) Show that $\displaystyle\int_0^1 \dfrac{dx}{\left(1 - x^n\right)^{\frac{1}{2}}} = \dfrac{\sqrt{x}\,\Gamma\left(\dfrac{1}{n}\right)}{n\,\Gamma\left(\dfrac{1}{n} + \dfrac{1}{2}\right)}$.

SECTION – C   (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

16. a) Prove that the matrices A, B, C given below have the same characteristic values.

$$A = \begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & b & a \\ b & 0 & c \\ a & c & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 & c & b \\ c & 0 & a \\ b & c & 0 \end{bmatrix}.$$

[OR]   **[K5]**

b) Show that $\begin{bmatrix} \dfrac{1+i}{2} & \dfrac{-1+i}{2} \\ \dfrac{1+i}{2} & \dfrac{1-i}{2} \end{bmatrix}$ is unitary.

17. a) Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that $1 - \sqrt{5}$ is a root.

[OR]   **[K4]**

b) Diminish by 2 the roots of the equation $x^4 + x^3 - 3x^2 + 2x - 4 = 0$.

18. a) Sum to infinity the series $\dfrac{1^2}{1!} + \dfrac{1^2 + 2^2}{2!} + \dots + \infty$.

[OR]   **[K5]**

b) Sum to infinity the series $\dfrac{7}{72} + \dfrac{7.28}{72.96} + \dfrac{7.28.49}{72.96.120} + \dots + \infty$.

19. a) Solve the following system by Gauss elimination method.

$10x + y + z = 12$; $2x + 10y + z + w = 13$; $x + y + 5z = 7$.

[OR]   **[K5]**

b) Solve the following system by Gauss Jordan elimination method.

$3x + 4y + 5z = 18$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$.

20. a) Evaluate $\displaystyle\int_0^\infty \dfrac{x}{1 + x^6}\,dx$.   **[OR]**   **[K4]**

b) Show that $\displaystyle\int_0^\infty \dfrac{dx}{\left(e^x + e^{-x}\right)^n} = \dfrac{1}{4} \beta\left(\dfrac{n}{2}, \dfrac{n}{2}\right)$ and evaluate $\displaystyle\int_0^\infty \sec h^6 x \, dx$.

****