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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2024 ONLY)

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REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: I **TIME : 3 HOURS**

PART - III

MATHEMATICAL STATISTICS-I

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. If a is a constants, then $E(ax) =$ ______ .

   (a) 1 (b) a (c) $aE(1)$ (d) $aE(x)$

2. ____ is the Var(X) for the joint probability density function $f(x,y) = \begin{cases} 2 - x - y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & elsewhere \end{cases}$

   (a) $\frac{11}{144}$ (b) $\frac{11}{12}$ (c) $\frac{1}{144}$ (d) $\frac{11}{14}$

3. Binomial distribution is applied for ______.

   a. continuous random variable b. discrete random variable
   c. irregular random variable d. uncertain random variable

4. For a ______ its mean, median and mode are equal.

   a. Binomial distribution b. Poisson distribution
   c. normal distribution d. moment generation function

5. The Mgf of Gamma distribution is ___

   (a) $np\sum_{x=1}^{n}\binom{n-1}{x-1}p^{x}q^{n-x}$ (b) $M_X(t) = (q + pe^t)^n$

   (c) $\sum_{r=0}^{\infty}\left(\frac{t}{\theta}\right)^r$ (d) $M_X(t) = (1-t)^{-\lambda}$, $|t| < 1$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define discrete random variables.
7. Define covariance
8. Define the moment generating function of Binomial distribution
9. Write the characteristic function of Rectangular distribution.
10. Define Exponential distribution.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Analyze the geometric mean G of the distribution $dF = 6(2-x)(x-1)dx$, $1 \le x \le 2$ is $6\log(16G) = 19$.

(OR)

b) For the frequency distribution $f(x)dx = \frac{2x}{9}dx$, $0 \le x \le 3$ find the mean and the standard deviation.

(CONTD…..2)

12.a) A random variable X assumes any positive integral value n with a probability to $\frac{1}{3^n}$. Find the expectation of X .

(OR)

b) Let $X_1, X_2, X_3, \ldots\ldots \ldots\ldots \ldots X_n$ be a random variables then prove that

$$V\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i^2 V(X_i) + 2\sum_{i=1}^{n}\sum_{\substack{j=1 \\ i<j}}^{n} a_i a_j Cov(X_i, X_j)$$

13.a) If X and Y are independent .Poisson variates with means λ_1 and λ_2 respectively, compute the probability that $(i)\ X + Y = k\ \ (ii)\ X = Y$.

(OR)

b) State and prove Chebychev's inequality .

14.a) Show that for the rectangular distribution $f(x) = \frac{1}{2a},\ -a < x < a$ the mgf about origin is $\frac{1}{at}\sinh at$ .Also show that moments of even order are given by $\mu_{2n} = \frac{a^{2n}}{(2n+1)}$.

(OR)

b) Derive moment generating function of Normal Distribution.

15.a) Derive the characteristic function of Gamma distribution.

(OR)

b) Discuss about Beta distribution.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) A random variable X has the following probability function

X	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

(i) Find k

(ii) Evaluate (a) P(X<6) (b) P(x≥6) (c) P(0<x<5)

(iii) If P (x≤ k) >$\frac{1}{2}$), find the minimum value of k

(iv) Determine the distribution function of X.

(OR)

b) For the following probability distribution $dF = y_0 e^{|x|} dx\ \ ,\ -\infty \le x \le \infty$ show that $y_0 = \frac{1}{2},\ \mu_1' = 0\ ,\ \sigma = \sqrt{2}$ and mean deviation about mean 1.

17.a) Three urns contain respectively 3 green and 2 white balls , 5 green and 6 white balls and 2 green and 4 white balls. One ball is drawn from each urn. Calculate the expected number of white balls drawn out.

(OR)

. (CONTD…..3)

17 b) Two random variables X and Y have the following joint probability density function:

$$f(x, y) = \begin{cases} 2 - x - y; & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

Calculate

(i) Marginal probability density function of X and Y

(ii) Conditional density functions

(iii) Var(X) and Var(Y)

(iv) Co-variance between X and Y

18. a) Fit a Poisson distribution to the following data which gives the number of doddens in a sample of clover seeds.

No of doddens(x)	0	1	2	3	4	5	6	7	8
Observed frequency (f)	56	156	132	92	37	22	4	0	1

(OR)

b) Seven unbiased coins are tossed and number of heads noted. The experiment is repeated 128 timers and the following distribution is obtained:

No of heads	0	1	2	3	4	5	6	7	Total
Frequencies	7	6	19	35	30	23	7	1	128

Reframe the binomial distribution for assuming

(i) The coin is unbiased.

(ii) The nature of the coin is not known.

19.a) If X is a normal variate with mean 30 and S.D 5. Evaluate the following probabilities (i) $26 \le X \le 40$ (ii) $X \ge 45$ (iii) $|X - 30| > 5$

(OR)

b) If x is normally distributed with mean 8 S.D 4 Evaluate (i) $P(5 \le X) \le 10)$ (ii) $P(10 \le X \le 15)$ (iii) $P(x \ge 15)$ (iv) $P(X \le 5)$

20.a) Derive the constants of Beta distribution of first kind.

(OR)

b) Derive the constants of Beta distribution of second kind.

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