(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2024 ONLY)

24UMS102

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024
COURSE NAME: B.Sc. -MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: I **TIME : 3 HOURS**

PART - III

CALCULUS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. A solution obtained by giving particular values to the arbitrary constants in a complete integral is called ____
   (a) Particular integral (b) general integral (c) singular integral (d) linear integral

2. Elimination of a and b from $z = (x+a)(y+b)$
   (a) $z = ab$ (b) $z = a + b$ (c) $z = p + q$ (d) $z = pq$

3. $\int_0^2 \int_1^2 (x - 3y^2)\,dy\,dx$ = __________ ___
   (a) -12 (b) 12 (c) 21 (d) -21

4. In Jacobians, u and v are independent variable then $\frac{\partial u}{\partial v}$ = _____ .
   (a) 1 (b) 2 (c) -1 (d) 0

5. _________ $= \frac{1}{s-a}$.
   (a) $L\left[e^{at}\right]$ (b) $L\left[e^{-at}\right]$ (C) $L(\sin at)$ (d) $L(\cos at)$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. P.I for $(D^2 + 16)y = \cos 4x$ is _____.
7. What is known as Lagrange's equation?
8. The value of $\int_0^a \int_0^a \int_0^a dx\,dy\,dz$ is_________.
9. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
10. $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ = _________ — (if s > 0).

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Solve $(D^2 + 4D + 4)y = e^{2x} - e^{-2x}$

(OR)

   b) Find the envelope of the family of a straight lines $y + tx = 2at + at^3$, the parameter being t

(CONTD………2)

12.a) Form a partial differential equation by eliminating the arbitrary function f from $f(x^2+y^2+z^2, z^2-2xy)=0$

(OR)

b) Solve the equation $q = xp + p^2$

13.a) Calculate $\int\int\int xyz$ dx dy dz taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

(OR)

b) Evaluate $\iint_R (x+y)^2\,dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

14.a) Estimate

(i) $\int_0^1 x^7(1-x)^8\,d\theta$

(ii) $\int_0^{\pi/2} \sin^7\theta\cos^5\theta\;d\theta$

(iii) $\int_0^{\pi/2} \sin^{10}\theta\;d\theta$

(OR)

b) Prove that $\beta(m,n) = \beta(m)\beta(n)$.

15.a) Evaluate $(i)\ L^{-1}\{\frac{1}{\sqrt{s}}\}$ $(ii)\ L^{-1}\{\frac{s}{s^2+16}\}$

(OR)

b) Find $L[\sin^3 2t]$

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Show that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$ is $4a\ \cos\frac{\theta}{2}$

(OR)

b) Solve $(D^3-2D+4)y=e^x\cos x$.

17. a) Solve $px(y^2+z)-qy(x^2+z)=z(x^2-y^2)$. Find the surface that contains the straight line $x+y=0, z=1$

(OR)

b) Solve the equation $(y+z)p+(z+x)q=x+y$

18. a) Evaluate the centroid of a loop of the lemniscates $r^2=a^2\cos 2\theta$

(OR)

b) By transforming into polar coordinates evaluate $\iint \frac{x^2y^2}{x^2+y^2}dx\ dy$ over the annular region between the circles $x^2+y^2=a^2\ and\ x^2+y^2=b^2\ (b>a)$ **(CONTD………3)**

19.a) Prove that $\iiint \frac{dxdydz}{(1-x^2-y^2-z^2)^{1/2}} = \frac{\pi^2}{8}$, the integration extended to all positive values of the variables for which the expression its real

(OR)

b) Summarize that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

20.a) Using Laplace transform, Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = \sin t,$ given that $y = \frac{dy}{dt} = 0$ where t=0

(OR)

b) Prove that

(i) $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t} dt = \log 2$

(ii) $L[\frac{\cos 3t - \cos 2t}{t}] = \frac{1}{2}\log(\frac{s^2+4}{s^2+9})$

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