(FOR THE CANDIDATES ADMITTED

 DURING THE ACADEMIC YEAR 2024 ONLY)

24UMS101

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2024
COURSE NAME: B.Sc.- MATHEMATICS　　　　　**MAXIMUM MARKS: 75**
SEMESTER: I　　　　　　　　　　　　　　　**TIME : 3 HOURS**

PART - III

CLASSICAL ALGEBRA

SECTION – A　　　　**(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS.　　　　　　　　　　**(K1)**

1.　One of the root of the equation $x^3 - 2x - 5 = 0$ lies between _____
　　(a) 2 and 3　　　　(b) -4 and 0　　　　(c) 3 and 4　　　　(d) 0 and 1

2.　$(1 + x)^n =$ _____ .

　　(a) $1 - nx + \dfrac{n(n-1)}{2!}x^2 - \dfrac{n(n-1)(n-2)}{3!}x^3 + \ldots$.

　　(b) $1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \ldots$.

　　(c) $1 - nx + \dfrac{n(n+1)}{2!}x^2 - \dfrac{n(n+1)(n+2)}{3!}x^3 + \ldots$.

　　(d) $1 + nx + \dfrac{n(n+1)}{2!}x^2 + \dfrac{n(n+1)(n+2)}{3!}x^3 + \ldots$.

3.　A function of the roots of an equation, which remains unaltered when any two of the roots are interchanged is called ____of the roots

　(a) Constant function　　(b) rational coefficients　　(c) symmetric function　(d) irrational coefficient

4.　The difference between the numbers of changes of sign when _____are substituted in the series of Strum's function will give the number of real roots of the equation.
　　(a) $0 \; to \; \infty$　　　　(b) $0 \; to \; -\infty$　　　　(c) $-\infty \; \& \; \infty$　　　　(d)   all

5.　Characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is_____.

　(a) $x^2 + 5x + 2 = 0$　　　(b) $x^2 + 5x - 2 = 0$　　　(c) $x^2 - 5x + 2 = 0$　　(d) $x^2 - 5x - 2 = 0$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.　　　　**(K2)**
6.　Write the order of convergence of the Newton-Raphson method.

7.　Write the formula for $\dfrac{e^x + e^{-x}}{2}$

8.　Find the quotient and the remainder when $2x^6 + 3x^5 - 15x^2 + 2x - 4$ is divided by $x + 5$.

9.　State the rule for finding the multiple roots of an equation f(x)=0
10. Write the statement of Cayley Hamiltonian Theorem.

SECTION – B   **(5 X 5 = 25 MARKS)**
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Calculate the smallest positive root of $x^2 - \log_e \frac{x}{e} - 12 = 0$ by Regula-Falsi method

(OR)

  b) Solve $x^3 = 2x + 5$ for the positive root by iteration method.

12.a) Sum the series $1 + \dfrac{1+3}{2!} + \dfrac{1+3+3^2}{3!} + \dfrac{1+3+3^2+3^3}{4!} + \cdots \cdots \infty$.

(OR)

  b) Determine the coefficient of x^n in the expansion of $\dfrac{x+1}{(x-1)^2(x-2)}$

13.a) Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

(OR)

  b) Solve the equation $x^3 - 9x^2 + 108 = 0$.

14.a) Calculate the rational root of $2x^3 - x^2 - x - 3 = 0$ and hence complete the solution of the Equation.

(OR)

  b) Calculate the number of real roots of the equation $x^4 - 14x^2 + 16x + 9 = 0$

15.a) Prove that $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ are idempotent matrics.

(OR)

  b) Prove that $\dfrac{1}{2}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is an orthogonal matrix

SECTION – C   **(5 X 8 = 40 MARKS)**
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.
(K4 (Or) K5)

16. a) Evaluate eal root of the equation $x^3 - 3x + 1 = 0$ lying between 1 and 2 correct to three places of decimal by using bisection method.

(OR)

  b) Evaluate the real root of the equation $\cos x = 3x - 1$ correct to four decimal places using iteration method

17.a) Evaluate the sum to infinity of the series

  $\dfrac{1}{24} - \dfrac{1.3}{24.32} + \dfrac{1.3.5}{24.32.40} - \cdots \cdots$

(OR)

  b) Sum the series to infinity $\dfrac{1.4}{5.10} - \dfrac{1.4.7}{5.10.15} + \dfrac{1.4.7.10}{5.10.15.20} \cdots \cdots$

18. a) Prove that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.

(OR)

 b) If $a + b + c + d = 0$ Prove that $\dfrac{a^5 + b^5 + c^5 + d^5}{5} = \dfrac{a^2 + b^2 + c^2 + d^2}{2} \cdot \dfrac{a^3 + b^3 + c^3 + d^3}{3}$

19.a) The equation $x^3 - 3x + 1 = 0$ has a root between 1 and 2 . Calculate it to three places of decimals.

(OR)

 b) Evaluate the roots of the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$ by finding the rational

 roots.

20.a) Determine the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(OR)

 b) Let A be a square matrix then prove that
  (i) The sum of the Eigen values of A is equal to the sum of the diagonal elements(trace) of A
  (ii) Product of Eigen values of A is $|A|$

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