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**(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)**

**23PMS209**

**REG.NO. :**

**N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI**
**END-OF-SEMESTER EXAMINATIONS : MAY-2024**
**COURSE NAME: M.Sc.-MATHEMATICS** **MAXIMUM MARKS: 75**
**SEMESTER: II** **TIME : 3 HOURS**

## NUMERICAL ANALYSIS

### SECTION – A (10 X 1 = 10 MARKS)

**ANSWER THE FOLLOWING QUESTIONS.**
**MULTIPLE CHOICE QUESTIONS. (K1)**

1. The value of $x_1$ for the function $f(x) = 3x + \sin x - e^x = 0$ using Newton's Method, setting $x_0 = 0,$ is ___.
   (a) 0.37 (b) 0.47 (c) 0.39 (d) 0.33
2. The Crout reduction method transforms the coefficient matrix into lower-triangular $\times$ upper-triangular matrix, but upper-triangular matrix has _____ on its diagonal.
   (a) ones (b) zeros (c) contants (d) same non-zero values
3. _____ method can be used to derive formulas for higher-order derivatives based on evenly spaced data.
   (a) Backward difference (b) Symbolic operator
   (c) Forward difference (d) Central difference
4. The solution of the differential equation $\frac{dy}{dx} + 2x + y = 0 \quad y(0) = 1,$ by Euler's method with $h = 0.1,$ to get $y(0.40) =$ _____.
   (a) 0.81 (b) 0.76 (c) –0.81 (d) –0.76
5. If the condition is the value of the derivative of function $u,$ then it is called _____ condition.
   (a) Dirichlet (b) Neumann (c) mixed (d) Hessenberg

**ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)**

6. What is the order of convergence for Newton's method?
7. Summarize the Gaussian Elimination method applied to the system of equations.
8. Write the error term of Simpson's 1/3 rule to the data.
9. What is the formula for the second-order Runge-Kutta method?
10. What is meant by characteristic-value problems?

### SECTION – B (5 X 5 = 25 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**

11.a) Find the complex root of the function $f(x) = x^3 + 2x^2 - x + 5 = 0$ with $x_0 = 1 + i$ by the Newton's method.

**(OR)**

b) Explain the concepts of Newton's method with multiple roots.

12.a) Using LU method, solve the matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{bmatrix}$.

**(OR)**

**(CONTD…..2)**

12. b) Find the value of the determinant by using elementary row transformations to make it upper-triangular: $\begin{vmatrix} 1 & 4 & -2 & 3 \\ 2 & 2 & 0 & 4 \\ 3 & 0 & -1 & 2 \\ 1 & 2 & 2 & -3 \end{vmatrix}$.

13.a) What is the estimated value for derivative if $x = 4.1$ for the function $f(x) = x^2 - x + 1$, for $x=0,2,3,5,6$, using divided difference method.

**(OR)**

b) Use Romberg integration to find the integral of $e^{-x^2}$ between the limits of $a = 0.2$ and $b = 1.5$.

14. a) Find $y(0.4)$ to the differential equation $\frac{dy}{dx} = -2x - y, \quad y(0) = -1$ with $h = 0.1$, by using Adams-Mouton method.

**(OR)**

b) Using Range-Kutta fourth order method, find $y(0.5)$ for the differential equation $\frac{dy}{dx} = -2x - y, \quad y(0) = -1$ with $h = 0.1$.

15.a) Compare the accuracy of the finite-difference method with the shooting method on the second-order boundary value problem: $u'' = u, \quad u(1) = 1.17520, \quad u(3) = 10.01787,$ whose analytical solution is $u = \sinh(x)$.

**(OR)**

b) Find the eigen values and eigen vectors of a matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix}$, by using Power method.

**SECTION – C (5 X 8 = 40 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**

**(K4 (Or) K5)**

16. a) Using remainder theorem, solve the cubic polynomial $x^3 + x^2 - 3x - 3 = 0$ by finding the successive errors, start with $x = 2$.

**(OR)**

b) Find the quadratic factors of the polynomial $x^4 - 1.1x^3 + 2.3x^2 + 0.5x + 3.3 = 0$ .

17. a) Solve the system of equations $2x_2 + x_4 = 0, \quad 2x_1 + 2x_2 + 3x_3 + 2x_4 = -2,$ $4x_1 - 3x_2 + x_4 = -7, \quad 6x_1 + x_2 - 6x_3 - 5x_4 = 6,$ by Gaussian elimination method.

**(OR)**

b) Solve the linear system of equations $8x_1 + x_2 - x_3 = 8, \quad 2x_1 + x_2 + 9x_3 = 12, \quad x_1 - 7x_2 + 2x_3 = -4,$ by Gauss seidal method.

18. a) Apply Simpson's $\frac{1}{3}$ rule to evaluate the integral $e^{-x^2}$ over the interval 0.2 to 1.5 using 2, 4, 6, 8, 10 subdivisions until the value converge to five decimal places.

**(OR)**

b) Apply Simpson's $\frac{3}{8}$ rule to evaluate the integral $e^{-x^2}$ over the interval 0.2 to 1.5 using 3, 6, 9, 12 subdivisions until the value converge to five decimal places.

**(CONTD…..3)**

19.a) Illustrate to calculate $y(0.2), y(0.4)$ and $y(0.6)$ for $\frac{dy}{dx} = -2x - y, \quad y(0) = -1,$ by using Runge-Kutta-Fehlberg method with $h = 0.2.$ Find the error of $y(0.6)$.

**(OR)**

b) Find $y(0.5)$ for the differential equation $\frac{dy}{dx} = -2x - y, \quad y(0) = -1,$ by Milne's method by taking $h = 0.1.$

20.a) Solve $u'' = u, \quad u'(1) = 1.17520, \quad u'(3) = 10.01787,$ with the finite-difference method.

**(OR)**

b) Consider the homogenous second-order equation with homogenous boundary conditions: $\frac{d^2u}{dx^2} + kx^2 = 0, \quad u(0) = 0, \quad u(1) = 0,$ where $k^2$.is a parameter. Solve this equation by characteristic values of the parameter.

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