(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS207

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI
END-OF-SEMESTER EXAMINATIONS: MAY-2024
COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**
SEMESTER: II **TIME: 3 HOURS**

PARTIAL DIFFERENTIAL EQUATIONS

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. The Solution $z=(x+a)(y+b)$ represents the partial differential equation ________.

   (a) $z=\frac{p}{q}$ (b) $z=pq$ (c) $z=p+q$ (d) $z=p-q$

2. Check the validity of the following statements:
   Statement A: If the operator $F(D,D')$ is reducible, then the order in which the linear factors occur is important.
   Statement B: If v_1, v_2 are the solutions of the homogeneous linear partial differential equation $F(D,D')z=0$ then lv_1+mv_2 is also a solution, where l,m are constants.

   (a) A and B are true (b) B is true (c) A is true (d) A and B are false

3. Kernel $K(s,x)$ of the Fourier transform is___________.

   (a) $\frac{1}{\sqrt{2\pi}}e^{-isx}$ (b) $\frac{1}{\sqrt{2\pi}}e^{sx}$ (c) $\frac{1}{\sqrt{2\pi}}\cos sx$ (d) $\frac{1}{\sqrt{2\pi}}e^{isx}$

4. If f is continuous function prescribed on the boundary S of some finite region V, determine a function $\psi(x,y,z)$ such that $\nabla^2\psi=0$ within V and $\psi=f$ on S. This is known as _______.

   (a) interior Dirichlet problem (b) exterior Dirichlet problem

   (c) interior Neumann problem (d) exterior Neumann problem

5. In the one dimensional wave equation, the constant C^2 represents__________.

   (a) $\frac{Temperature}{mass}$ (b) $\frac{Gravity}{mass}$ (c) $\frac{Tension}{density}$ (d) $\frac{Tension}{mass}$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Write subsidiary equation of a first order linear partial differential equation of the form $F(x,y,z,p,q)=0$ solved by Charpit's method.

7. Find the complementary function of the PDE $(D^2-D'^2)z=0$.

8. Write one dimensional diffusion equation .

9. If $\psi=\frac{q}{|\vec{r}-\vec{R}|}$ find the value of $\frac{\partial\psi}{\partial x}$.(q –constant and $R=(x',y',z')$ $a\ fixed\ point$).

10. If $\theta=\frac{1}{\sqrt{t}}\exp\left(-\frac{x^2}{4kt}\right)$, find the value of $\frac{\partial\theta}{\partial t}$.

(CONTD…..2)

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Find a complete integral of the equation $zpq = p + q$.

(OR)

b) Solve $p^2x + q^2y = z$ by Jacobi method,

12. a) Show that $z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$ satisfies the differential equation $z_{xx} - z_{yy} = \frac{2z}{x}$.

(OR)

b) Solve : $\left(D^2 - D'^2\right)z = x - y$.

13. a) Find the solution of the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0$, $(-\infty < x < \infty, y \geq 0)$, Which satisfying The conditions: (i) z and its partial derivatives tend to zero as $x \to \pm\infty$

(ii) $z = f(x), \frac{\partial z}{\partial y} = 0$ on $y = 0$.

(OR)

b) Show that if the two dimensional harmonic equation $\nabla_1^2 \mathrm{V} = 0$ is transformed to plane polar coordinates r and θ defined by $x = rcos\theta$, $y = rsin\,\theta$ then $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0$.

14. a) A uniform circular wire of radius 'a' charged with electricity of line density 'e' surrounds grounded concentric spherical conductor of radius 'c'. Determine the electrical charge density at any point on the conductor .

(OR)

b) Find the potential function $\psi(x, y, z)$ in the region $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$ satisfying the conditions (i) $\psi = 0$, on $x = 0, x = a, y = 0, y = b, z = 0$

(ii) $\psi = f(x, y)$ on $z = c, 0 \leq x \leq a, 0 \leq y \leq b$.

15. a) Derive the d' Alembert's solution of the one-dimensional wave equation.

(OR)

b) Find the temperature in a sphere of radius 'a' when its surface is maintained at zero temperature and its initial temperature is $f(r, \theta)$.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Determine the complete integral of the equation $(p^2 + q^2)y = qz$.

(OR)

b) Check whether the equations $xp - yq = x$, $x^2p + q = xz$ is compatible or not. If it is compatible, analyze their solution.

17. a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + 2\frac{\partial^2 z}{\partial x \partial y} = 0$ to canonical form and hence solve it.

(OR)

b) Determine the solution of the equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$.

(CONTD…..3)

18. a) Determine the solution of the equation $r + 4s + t + rt - s^2 = 2$.

(OR)

b) Verify whether the solution $z(x,t) = \sum c_n \cos(nx + \varepsilon_n) e^{-n^2 kt}$ is the solution of one dimensional heat equation.

19. a) A rigid sphere of radius 'a' is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

(OR)

b) Construct the general form of the potential function of the family of equipotential surfaces $x^2 + y^2 + z^2 = c x^{\frac{2}{3}}$.

20. a) The points of trisection of a string are pulled aside through a distance 'k' on opposite sides of the position of equilibrium and the string is released from rest. Determine the expression for the displacement of the string at any subsequent time and show that the mid-point of the stirng always remains at rest.

(OR)

b) A thin membrane of great extent is released from rest in the position $z = f(x, y)$. Determine the displacement at any subsequent time.

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