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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS208

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY-2024

COURSE NAME: M.Sc.-MATHEMATICS MAXIMUM MARKS: 75

SEMESTER: II TIME: 3 HOURS

MECHANICS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. (K1)

1. Which one of the following equation represents D'Alembert's Principle?

   (a) $\sum_{i=1}^{N}\left(\vec{F}_i - m_i \ddot{\vec{r}}_i\right).\delta\vec{r}_i = 0$ (b) $\sum_{i=1}^{N}\left(\vec{F}_i + m_i \ddot{\vec{r}}_i\right).\delta\vec{r}_i = 0$

   (c) $\sum_{i=1}^{N}\left(\vec{F}_i + m_i\vec{r}_i\right)_i = 0$ (d) $\sum_{i=1}^{N}\left(\vec{F}_i . m_i \ddot{\vec{r}}_i\right).\delta\vec{r}_i = 0$

2. The Lagrange's equation of motion is__________.

   (a) $\frac{d}{dt}\left(\frac{\partial L}{\partial q_j}\right) - \frac{\partial L}{\partial q_j} = 0$ (b) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) + \frac{\partial L}{\partial q_j} = 0$

   (c) $\frac{d}{dt}\left(\frac{\partial L}{\partial q_j}\right) - \frac{\partial L}{\partial \dot{q}_j} = 0$ (d) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$

3. _____is the necessary and sufficient condition that $f(q_1, q_2, \dots, q_n)$ have a stationary value at q_0.

   (a) $\delta f > 0$ (b) $\delta f < 0$ (c) $\delta f = 0$ (d) $\delta f \neq 0$

4. Hamilton's principal function is ____________.

   (a) $S(q_0, q_1, t_0, t_1) = \int_{t_0}^{t_1} L\, dt$ (b) $S(q_0, q_1, t_0, t_1) = \int_{t_0}^{t_1} H\, dt$

   (c) $S(q_0, t_0) = \int_{t_0}^{t_1} H\, dt$ (d) $S(q_0, q_1, t_0, t_1) = \int_{t_0}^{t_1} p\dot{q}\, dt$

5. The transformation $= aq + bp$, $P = cq + dp$ to be canonical if _____________.

   (a) $ad - bc = 1$ (b)$ad - bc = 0$ (c)$ac - bd = 0$ (d)$ab - cd = 0$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define Rhenomic,

7. If the Kinetic energy and potential energy of a particle are $T = \frac{mr^2}{2}\left(\dot{\theta}^2 + \omega^2 \sin^2\theta\right)$ and $V = mgrcos\,\theta$ respectively. Find the equations of motion with respect to θ.

8. Define Hamiltonian function.

9. Write the Hamilton –Jacobi equation.

10. State Poison theorem.

(CONTD…..2)

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Obtain the relation between principle of work and kinetic energy.

(OR)

b) Derive the matrix form of the rotational kinetic energy with an angular velocity.

12. a) Derive the Lagrange's equation of motion for a non-holonomic constraints.

(OR)

b) Utilize Routhian function, derive the equations of motion for a holonomic system.

13. a) Find the stationary values of the function $f = z$, subject to the constraints $\phi_1 = x^2 + y^2 + z^2 - 4 = 0, \phi_2 = xy - 1 = 0$.

(OR)

b) By using Hamiltonian function, derive the Hamilton's canonical equations of motion.

14. a) Derive the Hamilton-Jacobi equation.

(OR)

b) Prove that any complete solution of the Hamilton-Jacobi equation leads to a solution of the Hamilton problem.

15. a) Consider the transformation $Q = \sqrt{e^{-2q} - p^2}$, $P = \cos^{-1}\left(pe^q\right)$. Use the Poisson bracket to show that it is canonical.

(OR)

b) Consider the transformation $Q = \sqrt{2q}e^t \cos p$, $P = \sqrt{2q}e^{-t} \sin p$. Show that the transformations is canonical and hence find the generating function.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) A particle of mass m is suspended by a massless wire of length $r = a + b \cos \omega t$, $(a > b > 0)$to form a spherical pendulum. Analyze the equations of motion.

(OR)

b) Find expressions for the kinetic energy and the generalized momenta of the system of particles using spherical polar coordinates.

17. a) Two particles are connected by a rigid massless rod of length l which rotates in a horizontal plane with constant angular velocity ω. Knife-edge supports at the two particles prevent either particle from having a velocity component along the rod, but the particles can slide without friction in a direction perpendicular to the rod. Determine the differential equations of motion. Also solve for x, y and the constraint force as functions of time if the centre of mass is initially at the origin and has a velocity $v_0$in the positive y direction.

(OR)

b) Consider the spherical pendulum of length l. Reduce this problem to quadratures and determine the integrals of motion.

(CONTD…..3)

18. a) Analyze the curve equation $y(x)$ between the origin O and the point (x_1, y_1) such that a particle starting from rest at O, and sliding down the curve without friction under the influence of gravity, will reach the end of the curve in a minimum time.

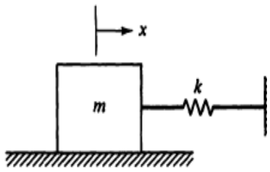
(OR)

b) A particle of mass m is attracted to a fixed point O by an inverse square force, $F_r = \frac{-\mu m}{r^2}$ where μ is the gravitational coefficient. Using the plane polar coordinates(r, θ) to describe the position of the particle. Examine the equations of motion using Hamiltonian function.

19. a) Consider the motion of the particle of unit mass which is attracted by an inverse square gravitational force to a fixed point O. Write the Kinetic energy and Potential energy for the particle in polar coordinates. Using Hamilton-Jacobi method to derive the eccentricity of an orbit.

(OR)

b) Consider the simple mass spring system. Using Hamilton-Jacobi equation, find the displacement and amplitude of the oscillation in x.



20. a) Consider the transformation $Q = log\left(\frac{\sin p}{q}\right)$ and $P = q \cot p$. Show that it is canonical. Determine the first two types of generating functions.

(OR)

b) Explain Poisson Brackets and its properties.

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