( NO. OF PAGES: 2 )

**(FOR THE CANDIDATES ADMITTED**

**DURING THE ACADEMIC YEAR 2022 ONLY)**

**22PMS417**

**REG.NO. :**

**N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI**

**END-OF-SEMESTER EXAMINATIONS : MAY-2024**

**COURSE NAME: M.Sc.-MATHEMATICS** **MAXIMUM MARKS: 50**

**SEMESTER: IV** **TIME : 3 HOURS**

# ALGEBRIC TOPOLOGY

## SECTION – A (10 X 1 = 10 MARKS)

**ANSWER THE FOLLOWING QUESTIONS.**

**MULTIPLE CHOICE QUESTIONS.** **(K1)**

1. A space X is said to be _______if the identity map $i_X : X \to X$ is nulhomotopic.
   a) contractible b) punctured c) homotopy d) quotient

2. If the set of all elements of the form $x^m$, for m belongs to Z, equals to G, then x is said to be a _____of G.
   a) cyclic b) generator c) order d) both a) and b)

3. Every n+1 by n+1 matrix with positive real entries has a_____eigen value.
   a) no b) negative c) positive d) both b) and c)

4. The closure of every cell in X meets only _____many open cells.
   a) finitely b) infinitely c) no d) none of these

5. Two cycles representing the same homology class are said to be _______
   a) homology group b) homologous c) boundaries d) chain complex

**ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.** **(K2)**

6. Define homomorphism induced by h.
7. Define covering space of B.
8. What is meant by homotopy equivalences?
9. Define linear metric.
10. What is meant by chain complex?

## SECTION – B (5 X 3 = 15 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)**

11. a) Show that the map $\hat{\alpha}$ is a group isomorphism.

**(OR)**

b) Show that the relations $\simeq$ *and* $\simeq_p$ are equivalence relations.

12.a) Let $p: E \to B$ be a covering map. If $B_0$ is a subspace of B, and if $E_0 = p^{-1}(B_0)$, then show that the map $p_0: E_0 \to B_0$ obtained by restricting p is a covering map.

**(OR)**

b) Examine that the fundamental group of $S^1$ is isomorphic to the additive group of integers.

**(CONTD….2)**

13.a) Applying the cocept of homotopy and show that the inclusion map $j: S^n \to \mathbb{R}^{n+1} - 0$ .induces an isomorphism of fundamental groups.

**(OR)**

b) State and prove Brouwer fixed-point theorem for the disc.

14.a) Examine that Every map $f: S^n \to S^{n+1}, k \geq 1$ is null homotopic.

**(OR)**

b) Show that, For $n \neq m, \mathbb{S}^n$ is not homotopy equivalent to $\mathbb{S}^m$.

15. a) If nonempty open sets $U \subset \mathbb{R}^m$ $and$ $V \subset \mathbb{R}^n$ are homeomorphic, then show that m=n.

**(OR)**

b) $\partial D^n$ is not a retract of $D^n$. Hence show that every map $f: D^n \to D^n$ has a fixed point.

**SECTION – C (5 X 5 = 25 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**

**(K4 (Or) K5)**

16. a) If X is a path connected and $x_0$ $and$ $x_1$ are two points of X, then prove that $\pi_1(X,x_0)$ is isomorphic to $\pi_1(X,x_1)$ .

**(OR)**

b) The operation * has the following properties: i) If [f]*([g]*[h]) is defined, so is ([f]*[g])*[h], and they are equal.

ii) "Given $x \in X$, let I denote the constant path X carrying all of I to the point x. If f is a path in X from $x_0$ $to$ $x_1$, then prove [f]*[$e_{x_1}$]=[f] and [$e_{x_0}$]*[f]=[f].

iii) Given the path f in X from $x_0$ $to$ $x_1$, let $\bar{f}$ be the path defined by $\bar{f}$ =f(1-s). it is called the reverse of f. then prove that [f]*[ $\bar{f}$ ]= [$e_{x_0}$] and [$\bar{f}$]*[f]= [$e_{x_1}$]".

17.a) Analyze the statement: the map $p: \mathbb{R} \to S^1$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map.

**(OR)**

b) Analyze the statement: Let $p: E \to B$ be a covering map, let $p(e_0) = b_0$. Any path $f: [0,1] \to B$ beginning at $b_0$ has a unique lifting to a path $\tilde{f}$ in E beginning at $e_0$.

18. a) State and prove the fundamental theorem of algebra.

**(OR)**

b) If $h: S^1 \to S^1$ is continuous and antipode-preserving, then prove that h is not nulhomotopic.

19.a) If K is a finite simplicial complex and $\mathfrak{U}$ is an open covering of $|K|$ then prove that there exists N such that for all $n \geq N$, $sd^n$ K is finer than $\mathfrak{U}$.

**(OR)**

b) State and prove Brouwer's invariance of domain theorem.

20.a) State and prove the five lemma.

**(OR)**

b) Prove that, the sequence of homology groups

$$\ldots \to H_n(A) \xrightarrow{i_*} H_n(B) \xrightarrow{j_*} H_n(C) \xrightarrow{\partial} H_{n-1}(A) \xrightarrow{i_*} H_{n-1}(B) \to \cdots$$ is exact.

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