**(FOR THE CANDIDATES ADMITTED** **23UAI2A1**

**DURING THE ACADEMIC YEAR 2023 ONLY)** **REG.NO. :**

**N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI**

**END-OF-SEMESTER EXAMINATIONS : MAY-2024**

**B.SC(CS WITH AI &ML)** **MAXIMUM MARKS: 75**

**SEMESTER: II** **TIME : 3 HOURS**

**PART - III**

**23UAI2A1- PROBABILITY AND STATISTICS**

**SECTION – A** **(10 X 1 = 10 MARKS)**

**ANSWER THE FOLLOWING QUESTIONS.** (K1)

1. If A and B are independent events then_____ independent event.
   (a) $\overline{A}$ and $\overline{B}$ (b) A and $\overline{B}$ (c) $\overline{A}$ and B (d) all
2. If $a$ is a constants, then $E(ax) =$ _______
   (a) 1 (b) a (c) $aE(1)$ (d) $aE(x)$
3. If two independent random variables have a bivariate normal distribution, they are independent iff ____
   (a) $\rho = 1$ (b) $\rho > 1$ (c) $\rho < 1$ (d) $\rho = 0$
4. If X has the standard normal distribution, then $X^2$ has the chi-square distribution with______ degrees of freedom .
   (a) $v = n$ (b) $v = 1$ (c) $v > 1$ (d) $v < n$
5. The correlation coefficient lies _____
   (a) $-1 \le \rho \le 1$(b) $0 \le \rho \le 1$ (c) $1 \le \rho \le \infty$ (d) $-1 \le \rho \le 0$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Define independent event
7. If X is the number of points rolled with a balanced die, find the expected value of $g(x) = 2X^2 + 1$
8. Define normal distribution.
9. Write the mean and variance of the finite population $\{c_1, c_2, \ldots\ldots\ldots, c_N\}$
10. Define standard error of estimate

**SECTION – B** **(5 X 5 = 25 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.** **(K3)**

11. a) Find the probabilities of randomly drawing two aces bin succession from an ordinary deck of 52 playing cards if we sample (i) without replacement (ii) with replacement

**(OR)**

b) A coin is tossed three times .Find the changes of throwing (i) three heads (ii) two heads and one tail (iii) head and tail alternatively.

( CONT….2)

12. a) If the probability density of X is given by $f(x)=\begin{cases}2(1-x) & for \quad 0<x<1 \\ 0 & elsewhere\end{cases}$

Then show that $E(X^r)=\dfrac{2}{(r+1)(r+2)}$ use this and evaluate $E[(2X+1)^2]$

**(OR)**

b) Given that X has the probability distribution $f(x)=\dfrac{1}{8}\binom{3}{x}$ $for \quad x=0,1,2 \; and \; 3$ find the moment generating function of this random variable and use it to be determine $\mu_1^{'}$ $and$ $\mu_2^{'}$

13. a) Prove that the moments generating function of the binomial distribution is $M_X(t)=[1+\theta(e^t-1)]^n$

**(OR)**

b) At a certain location on Highway I-10 , the number of cars exceeding the speed limit by more than 10 miles per hour in half an hour is a random variable having a Poisson distribution with $\lambda=8.4$. What is the probability of a waiting time of less than 5 minutes between cars exceeding the speed limit by more than 10 miles per hour?

14. a) If $X_r$ and $X_s$ are the r $^{th}$ and s $^{th}$ random variables of a random sample of size n drawn from the finite population $\{c_1,c_2,c_3,......,c_N\}$ then prove that $\operatorname{cov}(X_r,X_s)=-\dfrac{\sigma^2}{n-1}$

(OR)

b) For random samples of size n from an infinite population that has the value $f(x)$ $at$ $x$ ,the probability density of the r th order statistic $Y_r$ is given by

$$g_r(y_r)=\frac{n!}{(r-1)1(n-r)!}\left[\int_{-\infty}^{y_r} f(x)dx\right]^{r-1} f(y_r)\left[\int_{y_r}^{\infty} f(x)dx\right]^{n-r} \text{ for } -\infty<y_r<\infty$$

15. a) If the joint density of $X_1$ , $X_2$ and $X_3$ is given by $f(x_1,x_2,x_3)=\begin{cases}(x_1+x_2)e^{-x_3} \\ 0\end{cases}$

Find the regression equation on $X_2$ on $X_1$ and $X_3$

(OR)

b) Explain Multiple linear regression.

**SECTION – C (5 X 8 = 40 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.(K4 (Or) K5)**

16. a) Analyze the statement of Baye’s theorem.

**(OR)**

b) If A,B and C are any three events in a sample space S, then prove that
$P(A\bigcup B\bigcup C)=P(A)+P(B)+P(C)-P(A\bigcap B)-P(A\bigcap C)-P(B\bigcap C)+P(A\bigcap B\bigcap C)$

17. a) Prove that $\sigma^2=\mu_2^{'}-\mu^2$ ,make use of this to calculate the variance of X , representing the number of points rolled with a balanced die.

**(OR)**

b) State and prove Chebyshev’s theorem

( CONT….3)

18. a) Determine the mean and variance of the binomial distribution.

**(OR)**

b) Determine the mean and variance of the beta distribution.

19. a) State and prove central limit theorem.

**(OR)**

b) If $\overline{X}$ ans $S^2$ are the mean and variance of a random sample of size n from a normal population with the mean $\mu$ and the standard deviation $\sigma$, then prove that (i) $\overline{X}$ ans $S^2$ are independent (ii) The random variable $\frac{(n-1)S^2}{\sigma^2}$ has a chi-square distribution with n-1 degrees of freedom.

20. a) For the following data

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Hours studied x | 4 | 9 | 10 | 14 | 4 | 7 | 12 | 22 | 1 | 17 |
| Test score y | 31 | 58 | 65 | 73 | 37 | 44 | 60 | 91 | 21 | 84 |

Find the equation of the least squares line that approximates the regression of the test scores on the number of hours studied and predict the average test score of a person who studied 14 hours for the test.

**(OR)**

b) suppose that we want to determine on the basis of the following data whether there is a relationship between the time in minutes , it takes a secretary to complete a certain form in the morning and in the late afternoon:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Morning x | 8.2 | 9.6 | 7.0 | 9.4 | 10.9 | 7.1 | 9.0 | 6.6 | 8.4 | 10.5 |
| Afternoon y | 8.7 | 9.6 | 6.9 | 8.5 | 11.3 | 7.6 | 9.2 | 6.3 | 8.4 | 12.3 |

Compute and interpret the sample correlation coefficient.

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