(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2023 ONLY)

23UMS203

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY-2024

COURSE NAME: B.Sc.- MATHEMATICS MAXIMUM MARKS: 75

SEMESTER: II TIME : 3 HOURS

PART - III

TRIGONOMETRY, VECTORCALCULUS AND FOURIER SERIES

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **K1**

1. In the expansion of cos n θ, the terms are___________.
   a) alternately positive and negative b) positive
   c) negative d) none of the above

2. $\lim_{n\to\infty} \frac{1^2+2^2+3^2+4^2 \ldots\ldots n^2}{n^2}$ is equal to ______________.
   a) 1 b) 1/3 c) ½ d) ¼

3. When do you say the function f(x) is even?
   a) f(-x) = -f(x) b) f(-x) = f(x) c) f(x) = 0 d) f(x) = 1

4. The directional derivative of a scalar point function in the direction of $\vec{a}$ is given by $\boldsymbol{\nabla\varphi} \cdot \vec{a}\ / |\vec{a}|$ then the maximum value of directional derivative is ______________.
   a) 1 b) 0 c) ∇φ d) none of the above

5. The relation between a line integral to the double integral taken over the region bounded by the closed curve is given by ____________.
   a) Stock's theorem b) Green's theorem
   c) Gauss divergence theorem d) none of the above

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **K2**

6. True or False: "The sum of the powers of cosθ and sinθ in every term of the expansions equals n".
7. State the Cauchy root test.
8. If the function f(x) is an odd function, then what does its Fourier series have only?
9. When the two surfaces φ_1 and φ_2 are orthogonal?
10. State Stoke's theorem.

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **K3**

11. a) Express cos8θ in terms of sinθ.

(OR)

b) Prove that cos 8θ = 1-32 $\sin^2$ θ +160 $\sin^4$θ -256 $\sin^6$θ +128 $\sin^8$θ.

12. a) Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\cdots$

(OR)

b) Show that the series $1+\frac{1}{2}+\frac{1}{3}+\cdots$ is diverges, by using Cauchy Condensation test .

(CONTD…….2)

13. a) Express $f(x) = x - \pi$ as Fourier series in the interval $-\pi < x < \pi$.

(OR)

b) Find Fourier series of $f(x) = x$ in $-\pi< x <\pi$. Deduce that $\pi/4 = 1-(1/3)+(1/5)-(1/7)\ldots$.

14. a) Show that the vector $\vec{F} = (x+2y+az)\vec{i} + ((bx+3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational.

(OR)

b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point (2, -1, 2)

15. a) Find the work done, when a force $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ moves a particle from the origin to the point $(1, 1)$ along $y^2 = x$.

(OR)

b) Show that $\iint_S \vec{F} \cdot \hat{n}\, ds = 3/8$, where $\vec{F} = (y z\vec{i} + z x\vec{j} + x y\vec{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K3

16. a) Prove that $\sin^4\theta\cos^3\theta = \frac{1}{64}(\cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta)$.

(OR)

b) If $\tan(A + iB) = x + iy$. prove that

(i) $x^2+y^2+ 2x \cot 2A = 1$

(ii) $x^2 + y^2-2y \coth 2B + 1 =0$

(iii) $x \sinh 2B = y \sin 2A$

17. a) Examine the convergence of the series $\sum_1^\infty \left(\frac{n}{n+1}\right)^{1/2} x^n$.

(OR)

b) Test the convergence and divergence of the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \ldots$

18. a) Find Fourier series of $f(x) = x^2$ in $-\pi< x <\pi$: Hence deduce that

(i) $1-\frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots \ldots = \frac{\pi^2}{12}$

(ii) $1+\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \ldots \ldots = \frac{\pi^2}{6}$

(iii) $1+\frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{5^2} \ldots \ldots = \frac{\pi^2}{8}$

(OR)

b) Find the half range cosine and sine series of $f(x) = x$; $0 < x <\pi$. Hence deduce that

$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots = \frac{\pi^2}{8}$.

19. a) Prove that $\nabla^2(r) = f''(r) + (2/r)f'(r)$

(OR)

b) Prove that $div\,(grad\,r^n) = n(n+1)\, r^{n-2}$

20. a) Verify Stoke's theorem in a plane for $\vec{F} = (y - z + 2)\vec{i} - (yz + 4)\vec{j} - xz\vec{k}$ where S is the open surface of the cube formed by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$ above the xy – plane.

(OR)

b) Verify the Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} - yz\vec{k}$ over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

****

ETHICAL PAPER