(FOR THE CANDIDATES ADMITTED **(NO. OF PAGES: 2)**

DURING THE ACADEMIC YEAR 2021 ONLY) **21UMS615**

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS: MAY-2024
COURSE NAME: B.Sc.- MATHEMATICS **MAXIMUM MARKS: 70**
SEMESTER: VI **TIME : 3 HOURS**

PART - III

COMPLEX ANALYSIS

SECTION - A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **K1**

1. If a harmonic function $u(x,y)$ is given we can find another harmonic function $v(x,y)$ so that ____________.
(a) $u+v$ is analytic (b) $u+iv$ is analytic (c) $u+iv$ is not analytic (d) None of these

2. The form of $1+z+z^2+\cdots+z^{n-1}$ is__________.
(a) $\frac{1-z}{1-z^n}$ (b) $\frac{1-z^n}{1-z}$ (c) $\frac{1-z}{1+z^n}$ (d) None of these

3. The values of sin i is ___________.
(a) $i(1+\frac{1}{3!}+\frac{1}{5!}+\cdots)$ (b) $1+\frac{1}{3!}+\frac{1}{5!}+\cdots$ (c) $i(1-\frac{1}{3!}-\frac{1}{5!}+\cdots)$ (d) None of these

4. If $f(t) = u(t) + iv(t)$ is a continuous function, defined in an interval (a,b), then________.
(a) $\int_b^a f(t)dt = \int_b^a u(t)dt + i\int_a^b v(t)dt$ (b) $\int_a^b f(t)dt = \int_b^a u(t)dt - i\int_b^a v(t)dt$
(c) $\int_a^b f(t)dt = \int_a^b u(t)dt + i\int_a^b v(t)dt$ (d) none of these .

5. If $f(z)$ and $g(z)$ are analytic in Ω , and if $f(z)=g(z)$ on a set which has _________.
(a) Not an accumulation point in Ω. (b) an accumulation point in Ω
(c) an accumulation point in Ω'. (d) none of these

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **K2**

6. State Lucas's theorem.
7. Define convergent sequence.
8. Define A complex-valued function f(z).
9. Define Rectifiable Arcs.
10. Define isolated singularity of f(z).

SECTION – B **(5 X 4 = 20 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **K3**

11. a) If all zeros of a polynomial $P(z)$ lie in a half plane, then prove that all zeros of the derivative $P'(z)$ lie in the same half plane.

(OR)

b) Show that $f(z)=z^3$ is analytic.

(CONTD.......2)

12. a) Prove that a sequence is convergent if and only if it is a Cauchy sequence.

(OR)

b) Show that the limit function of a uniformly convergent sequence of continuous functions is itself continuous.

13.a) Use the addition formulas to separate $cos\ (x\ + iy)$, $sin\ (x\ + iy)$ in real and imaginary parts.

(OR)

b) Prove that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant. Also prove the same is true if either the real part, the imaginary part, the modulus, or the argument is constant.

14.a) Let f(z) be analytic on the set R' obtained from a rectangle R by omitting a finite number of interior points ζ_j. If it is true that $\lim_{z\to\zeta_j}(z-\zeta_j)f(z)=0$, for all j, then prove that $\int_{\partial R} f(z)dz = 0$.

(OR)

b) State and prove Cauchy's Theorem in a Disk

15.a) State and prove The maximum principle theorem.

(OR)

b) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS FROM Qn.No : 17 to 21) K4&K5

16. State and prove Cauchy's Theorem for a Rectangle.

17. State and prove Cauchy-Riemann Equation on necessary conditions for differentiability of a function.

18. If $\sum_0^\infty a_n$ converges, then prove that $f(z) = a_n z^n$ tends to $f(1)$ as z approaches 1 in such a way that $|1-z|/(1-|z|)$remains bounded.

19. Prove that addition theorem of the exponential function clearly implies.
(i) $\log(z_1 z_2) = \log z_1 \log z_2$ (ii) $\arg(z_1 z_2) = argz_1 + argz_2$

20. Show that the line integral$\int_\gamma\ pdx + qdy$, defined in Ω, depends only on the end points of γif und only if there exists a function $U(x,y)$ in Ω with the partial derivatives
$\partial U/_{\partial x} = p, \partial U/_{\partial y} = q$

21. Suppose that $f(z)$ is analytic in the region Ω' obtained by omitting a point a from a region Ω. Prove that a necessary and sufficient condition that there exist an analytic function in Ω which coincides with $f(z)$ in Ω' is that $\lim_{z\to a}(z-a)f(z) = 0$. The extended function is uniquely determined.

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ETHICAL PAPER