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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2021 ONLY)

21UMS614

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS :MAY-2024

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: VI **TIME : 3 HOURS**

PART - III

REAL ANALYSIS – II

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. (K1)

1. The value of c for the functions $f(x) = x, g(x) = x^2 (0 \leq x \leq 1)$ for which $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$ is______________.
   (a) 0 (b) 2 (c) $\frac{1}{2}$ (d) 1

2. The set of discontinuous of monotonic function f on [a, b] is_______________.
   (a) countable (b) uncountable (c) monotonic (d) continuous

3. If $f \in R(\alpha)$ and $f \in R(\beta)$ on [a, b], then which of the following is true ?
   (a) $f \in R(c_1\alpha + c_2\beta)$ (b) $f \in R(c_1\alpha - c_2\beta)$
   (c) $f \in R(c_1 + c_2)\beta$ (d) $f \in R(\frac{c_1\alpha}{c_2\beta})$

4. Which of the following is true for any two partitions P_1 and P_2 on [a, b]?
   (a) $L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$ (b) $L(P_1, f, \alpha) = U(P_2, f, \alpha)$
   (c) $L(P_1, f, \alpha) \geq U(P_2, f, \alpha)$ (d) $L(P_1, f, \alpha) < U(P_2, f, \alpha)$

5. Let α be of bounded variation on [a, b] and $f \in R(\alpha)$ on [a, b]. Then F(x) is ______.
   (a) $\int_a^b f\, d\alpha$ (b) $\int_a^x f\, d\alpha$ (c) $\int_x^b f\, d\alpha$ (d) $\int_x^a f\, d\alpha$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define Local minimum at a point on a subset S of a metric space.
7. Define total variation of the function .
8. Define step function .
9. Define upper and lower Stieltjes sum of the function.
10. State second fundamental theorem of integral calculus.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Assume f has a derivative at each point of an open interval (a, b), and assume that f is continuous at both end points a and b. If $f(a) = f(b)$ then prove that there is atleast one interior point c at which $f'(c) = 0$.

(OR)

b) State and prove Taylor's formula with remainder.

(CONTD….2)

12.a) If f is monotonic on [a, b], then prove that $f$is of bounded variation on [a, b].

(OR)

b) Let f be of bounded variation on [a, b]. Let V be defined on [a, b] as follows: $V(x) = V_f(a, x), a < x \leq b, V(a) = 0$. Prove the following

(i) V is an increasing function on [a, b].

(ii) V – f is an increasing function on [a, b].

13.a) If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on [a, b], prove that $c_1 f + c_2 g \in R(\alpha)$.

(OR)

b) State and prove Euler's summation formula.

14.a) If $P'$is finer than P and α is increasing on [a, b], then prove that $U(P', f, \alpha) \leq U(P, f, \alpha)$

(OR)

b) If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on [a, b] and α is increasing on [a, b] then prove that the product $f.g \in R(\alpha)$ on [a, b].

15.a) State and prove Second Mean – Value theorem for Riemann – Stieltjes integrals.

(OR)

b) If $f \in R$ and $g \in R$ on [a, b], let $F(x) = \int_a^x f(t)dt$, $G(x) = \int_a^x g(t)dt$ then prove that F and G are continuous functions of bounded variation on [a, b].

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. If f is differentiable at c and if g is differentiable at $f(c)$ then prove that $g \circ f$ is differentiable at c and $(g \circ f)'c = g'[f(c)]f'(c)$.

17. State and prove Intermediate value theorem for derivative.

18. Let f be of bounded variation on [a, b]. If $x \in (a, b]$, let $V(x) = V_j(a, x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V.

19. If $f \in R(\alpha)$ on [a, b] and if α has a continuous derivative α' on [a, b]. Prove that the Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists and $\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.

20. If α is increasing on [a, b]. Prove that the following are equivalent:
(i) $f \in R(\alpha)$on [a, b].
(ii) f satisfies Riemann's condition with respect to α on [a, b].
(iii) $\underline{I}(f, \alpha) = \overline{I}(f, \alpha)$.

21. Ifg has a continuous derivative g' on an interval [c, d]. Let f be continuous on $g([c, d])$ And define F by the equation $F(x) = \int_{g(c)}^x f(t)dt$if $\in g([c, d])$.
Prove that for each x in [c. d] the integral $\int_c^x f[g(t)]g'(t)dt$ exists and
$\int_{g(c)}^{g(d)} f(x)dx = \int_c^d f[g(t)]g'(t)dt$

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