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**(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)**

**23PMS104**

**REG.NO. :**

**N.G.M. COLLEGE (AUTONOMOUS): POLLACHI**
**END-OF-SEMESTER EXAMINATIONS: NOVEMBER-2023**
**COURSE NAME: M.Sc.- MATHEMATICS** **MAXIMUM MARKS: 75**
**SEMESTER: I** **TIME: 3 HOURS**

## ORDINARY DIFFERENTIAL EQUATIONS

### SECTION – A (10 X 1 = 10 MARKS)

**ANSWER THE FOLLOWING QUESTIONS.**
**MULTIPLE CHOICE QUESTIONS.** **(K1)**

1. If two differentiable functions $x_1$ and $x_2$ defined on I are linearly dependent on *I* then Value of Wronskin is _____on *I*.

   a) $W[x_1(t), x_2(t)] \neq 0$ b) $W[x_1(t), x_2(t)] = 0$ c) both (a) and (b) d)none of these

2. A point $a \in I$ is called _____ Point.
   a) Particular b) Ordinary c) Numerical d) Complex

3. $\Phi(t)$ is a _____ matrix.
   a) Fundamental b)Non-singular c) Row d) Square

4. . Lipschitz condition is defined as _____.

   a) $|f(t, x_1) - f(t, x_2)| \leq K|x_1 - x_2|$ b) $|f(t, x_1) + f(t, x_2)| \leq K|x_1 + x_2|$

   c) $|f(t, x_1) - f(t, x_2)| \geq K|x_1 - x_2|$ d) $|f(t, x_1) + f(t, x_2)| \geq K|x_1 + x_2|$

5. A linear BVP which is not regular is called _____linear BVP,
   a)Singular b)Periodic c)Regular d)Irregular

**ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.** **(K2)**

6. Define Linear independence.
7. Define analytic function.
8. Define fundamental Matrix.
9. Define extremal solutions.
10. Define periodic boundary conditions.

### SECTION – B (5 X 5 = 25 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.** **(K3)**

11. a) Solve $x'' - 9x' + 20x = 0 \; ; -\infty \prec t \prec \infty.$

**(OR)**

b) Solve the IVP $x'' + x' + 2x = 0, \; x(0) = 0, x'(0) = 2.$

12.a) Prove that if $P_n(t)$ and $P_m(t)$ are Legendre polynomials, then $\int_{-1}^{1} P_n(t)P_m(t)\,dt = 0$ if $m \neq n.$

**(OR)**

b) Prove that $\frac{d}{dt}\left[t^p J_p(t)\right] = t^p J_{p-1}(t).$

**(CONTD….2)**

13. a) Determine a fundamental matrix for $x' = Ax$, where $A = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$,

**(OR)**

b) Find the solution for a system of equations $x_1' = 5x_1 - 2x_2$, $x_2' = 2x_1 + x_2$

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14. a) Prove that IVP $x' = -x, x(0) = 1, t \geq 0.$ is equivalent to the integral equation

$$x(t) = 1 - \int_{t_0}^{t} x(s)ds.$$

**(OR)**

b) State and prove Gronwall inequality.

15. a) Solve the BVP: $x'' + |x| = 0, 0 \leq t \leq a \prec \pi, x(0) = 0, x(a) = 1.$

**(OR)**

b) Solve the BVP: $x'' + \cos x = 0, x(0) = x(1) = 0.$

## SECTION – C (5 X 8 = 40 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**

**(K4 (Or) K5)**

16. a) Solve $x''' - 6x'' + 11x' + 6x = 0 \; ; -\infty \prec t \prec \infty.$

**(OR)**

b) Solve the IVP $x''' + x'' = 0, \; x(0) = 1, x'(0) = 0, x''(0) = 1.$

17.a) Prove that if $P_n$ are Legendre polynomials, then $\int_{-1}^{1} P_n^2(t)dt = \frac{2}{2n+1}.$

**(OR)**

b) Prove that $\frac{d}{dt}\left[t^{-p} J_p(t)\right] = -t^{-p} J_{p+1}(t).$

18. a) State and Prove Existence and Uniqueness theorem.

**(OR)**

b) Find $e^{At}$ $when$ $(i)\, A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ $(ii)\, A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$

19. a) State and Prove Picard's Theorem.

**(OR)**

b) State and Prove Existence Theorem.

20.a) If *G(t,s)* is given by the relation $\begin{cases} -y(t)\,z(s)/A & if \;\; t \leq s, \\ -y(s)\,z(t)/A & if \;\; t \geq s. \end{cases}$ Prove that x(t) is a solution of $L(x) + f(t) = 0, \; a \leq t \leq b$ if and only if $x(t) = \int_{a}^{b} G(t,s)\, f(s)ds.$

**(OR)**

b) Solve that BVP $x'' + \lambda x = 0, x(0) = 0, x'(1) = 0$

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