(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS103

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

COURSE NAME: M.Sc.- MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: I **TIME : 3 HOURS**

COMPLEX ANALYSIS

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. A cycle γ is an open set Ω is said to be homologous to zero with respect to Ω if ___________ for all points 'a' in the complement of Ω
   (a) n(γ ,a) = 0 (b) n(γ ,a) = 0 (c) n(γ ,a) < 0 (d) n(γ ,a) > 0
2. A real valued function u(z) is said to be harmonic if it satisfies __________.
   (a) elliptic equation (b) Laplace equation (c) parabolic equation (d) hyperbolic eqation
3. Convergence is uniform on every _________.
   (a) closed subset (b) open subset (c) compact subset (d) none of these
4. The compact subsets of C are the ___________.
   (a) bounded and open sets (b) bounded and closed sets
   (c) unbounded and open sets (d) unbounded and closed sets
5. A function f(z) is said to be periodic with period w if ____________.
   (a) f(z+w) = f(z) (b) f(z-w) = f(z) (c) f(zw) = f(z) (d) f(z/w) = f(z)

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. What is the nature of f if f(z) is analytic in the whole plane and if $z^{-1}\,\mathrm{Re}\,f(z) \to 0$ when $z \to \infty$?
7. State Cauchy's residue theorem.
8. What is the value of $\Gamma\left(\frac{1}{2}\right)$?
9. What is the use of Jensen formula?
10. What is the sum of the residues of an elliptic function?

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **(K3)**

11. a) Prove that a region Ω is simply connected iff n(γ ,a) = 0 for all cycles γ in Ω and all points 'a' which do not belong to Ω.

(OR)

b) State and prove Rouche's theorem.

12. a) State and prove the Mean-value property.

(OR)

b) Derive Poisson formula.

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13. a) State and prove Hurwitz theorem.

(OR)

b) Prove that every function which is meromorphic in the whole plane is the quotient of two entire functions.

14.a) Prove that a family F of analytic functions is normal with respect to C if and only if the functions in F are uniformly bounded on every compact set.

(OR)

b) Prove that a locally bounded family of analytic functions has locally bounded derivatives.

15.a) Prove that an elliptic function without poles is a constant.

(OR)

b) Prove that any two bases of the same module are connected by a unimodular transformations.

SECTION – C (5 X 8 = 40 MARKS) (K4 (Or) K5)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

16. a) If pdx + qdy is locally exact in Ω, prove that $\int_{\gamma} pdx + qdy = 0$ for every cycle $\gamma \approx 0$ in Ω. **K4**

(OR)

b) If $R(\cos\theta, \sin\theta)$ is a rational function in $\cos\theta$ and $\sin\theta$, derive the formula to find $\int_0^{2\pi} R(\cos\theta, \sin\theta)d\theta$. Find also $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}, a > 1$. **K5**

17. a) State and prove Schwarz theorem. **K4**

(OR)

b) State and prove the reflection principle. **K4**

18. a) State and prove Mittag-Leffler theorem. **K4**

(OR)

b) Prove that $\sin \pi z$ is a entire function of genus 1. **K5**

19. a) State and prove Jensen’s formula and hence deduce the Poisson-Jensen formula. **K4**

(OR)

b) State and prove Arzela-Ascoli theorem. **K4**

20. a) Prove that there exists a basis (w_1, w_2) such that the ratio $r = w_2 / w_1$ satisfies the following conditions: (i) $\operatorname{Im} r > 0$, (ii) $-1/2 < \operatorname{Re} r \leq 1/2$, (iii) $|r| \geq 1$, (iv) $\operatorname{Re} r \geq 0$ if $|r| = 1$. Also prove that the ratio r is uniquely determined by these conditions, and there is a choice of two, four, or six corresponding bases. **K5**

(OR)

b) Prove that the zeros $a_1, \ldots, a_n$ and poles $b_1, \ldots, b_n$ of an elliptic function satisfy $a_1 + \ldots + a_n \equiv b_1 + \ldots + b_n \ (Mod\ M)$. **K4**

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