(NO. OF PAGES: 3)

(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2023 ONLY)

23PMS102

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER: I **TIME : 3 HOURS**

REAL ANALYSIS

SECTION – A **(10 X 1 = 10MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. The partition P* is a refinement of P if ______________

   a) $P \subset P^*$ b) $P \supset P^*$ c) $P = P^*$ d) $P \neq P^*$

2. Every uniformly convergent sequence of bounded functions is __________

   a) bounded b) uniformly bounded c) continuous d) uniformly continuous

3. dim R^n = ________

   a) 0 b) n c) 1 d) ∞

4. The outer measure of an interval is its _______

   a) least value b) maximum value c) length d) average

5. max{f(x),0} is the function__________.

   a) f(x) b) $f^-(x)$ c) $f^{-1}(x)$ d) $f^+(x)$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define a closed curve.
7. Define a pointwise bounded sequence.
8. Define the norm ||A||.
9. Give the canonical representation of the simple function ϕ.
10. Define the lower Lebesgue integral.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that $f \in R(\alpha)$ on [a,b] iff for every $\epsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L\,(P, f, \alpha) < \epsilon$.

(OR)

b) Establish and Prove integration by parts rule.

12. a) Give an example to show that the limit of the derivative need not be the derivative of the limit.

(OR)

b) Show by an example that the uniformly bounded convergence sequence of functions even if defined on a compact set need not contain uniformly convergent subsequence.

(CONTD…….2)

13.a) Prove that a linear operator A on a finite dimensional vector space is one-to-one if and only if the range of A is all of X.

(OR)

b) Suppose E is an open set in R^n, f maps E into R^m, f is differentiable at $x_0 \in$ E, g maps an open set containing f(E) into R^k, and g is differentiable at $f(x_0)$. Then the mapping F of E into R^k defined by F(x) = g(f (x)) is differentiable at x_0, and $F'(x_0) = g'(f(x_0))f'(x_0)$.

14.a) Prove that any set of outer measure zero is measurable. In particular, any countable set is measurable.

(OR)

b) Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise a.e. on E to the function f. Then show that f is measurable.

15. a) Let f be a bounded measurable function on a set E of finite measure. Then prove that f is integrable over E.

(OR)

b) State and prove the integral comparison test.

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

. (K4 (Or) K5)

16. a) Assume α increases monotonically and $\alpha' \in R$ on [a, b]. Let f be a bounded real function on [a, b].Then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. Prove also that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

(OR)

b) If γ' is continuous on [a, b], then prove that γ is rectifiable, and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)|dt.$$

17. a) Suppose K is compact, and

(i) $\{f_n\}$ is a sequence of continuous functions on K,

(ii) $\{f_n\}$ converges pointwise to a continuous function f on K,

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, n = 1, 2, 3, ...

Prove that $f_n \to f$ uniformly on K.

(OR)

b) State and Prove the Stone Weierstrass Theorem.

18. a) If X is a complete metric space, and if φ is a contraction of X into X, then show that there exists one and only one $x \in X$ such that $\varphi(x) = x$.

(OR)

b) State and Prove the Inverse Function Theorem.

19. a) Prove that every interval is measurable.

(OR)

b) State and prove the simple approximation theorem.

20. a) Let $\{f_n\}$ be a sequence of measurable functions on a set of finite measure E. Suppose $\{f_n\}$ is uniformly pointwise bounded on E, that is, there is anumber $M \geq 0$ for which $|f_n| \leq M$ on E for all n.If $\{f_n\} \longrightarrow f$ pointwise on E, then prove that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

(OR)

b) State and prove the Lebesgue Dominated Convergence Theorem.

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