

(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS312

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER - 2023

COURSE NAME: M.Sc.- MATHEMATICS

MAXIMUM MARKS: 50

SEMESTER: III

TIME : 3 HOURS

FUNCTIONAL ANALYSIS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

(K1)

MULTIPLE CHOICE QUESTIONS.

1. Every finite dimensional subspace is _____ with respect to any norm on it.
a) open b) closed c) convex d) complete
2. Every normed linear space with a denumerable basis is _____.
a) continuous b) separable c) denumerable d) dense
3. Any two finite dimensional normed linear spaces of the same dimension are linearly _____.
a)homeomorphic b) homomorphic c) isometric d) continuous
4. If X be a normed linear space and $x \in X$ then there exists $f \in X'$ such that _____.
a) $f(x) = \|x\|, \|f\| = 1$ b) $f(x) \neq \|x\|$ c) $f(x) \leq \|x\|, \|f\| = 1$ d) $f(x) \geq \|x\|$
5. Let X and Y be normed linear spaces and $A: X \rightarrow Y$ be a linear operator, if A is an open map then A is _____.
a)injective b) surjective c) bijective d) continuous

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.

(K2)

6. Write the Polarization Identity for $\mathbb{K} = \mathbb{C}$.
7. State the Riesz lemma.
8. Write the Fourier expansion formula.
9. Define complementary subspace.
10. State the Bounded inverse theorem.

SECTION – B

(5 X3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) State & Prove Cauchy-Schwarz inequality in K^n .

(OR)

- b) Prove that if X_0 is complete subspace of a normed linear space X and X/X_0 is a Banach space then X is a Banach space.

(CONTD.....2)

12. a) Prove that interior of a proper subspace of a normed linear space is empty.
(OR)
b) State and prove the Projection theorem.
- 13.a) If X is a Hilbert space then prove that for every continuous functional f on X , there exists a unique $v \in X$ such that $f(x) = \langle x, v \rangle \forall x \in X$.
(OR)
b) State and prove the Riesz-Fischer theorem.
- 14.a) Let X and Y be normed linear spaces and $A \in \mathcal{B}(X, Y)$ Prove that $\|A'\| = \|A\|$.
(OR)
b) Let X and Y be normed linear spaces such that $X \neq \{0\}$ and let $B(X, Y)$ be a Banach space then Prove that Y is a Banach space.
- 15.a) Let X be a Banach space, Y be a normed linear space, and $A \subseteq B(X, Y)$. If A is pointwise bounded then A is uniformly bounded.
(OR)
b) State & Prove Open mapping theorem.

SECTION – C**(5 X 5 = 25 MARKS)****ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)**

16. a) State and Prove Minkowski's Inequality in $\mathcal{F}(\mathbb{N}, \mathbb{K})$.
(OR)
b) Let X be a finite dimensional normed linear space, $E = \{u_1, \dots, u_k\}$ be a basis of X and the norm defined by $\|x\|_E = \max\{|f_1(x)|, \dots, |f_k(x)|\}$, $x \in X$. Prove that any two norms on a finite dimensional linear space are equivalent.
17. a) State and Prove Baire category theorem.
(OR)
b) State and Prove Heine-Borel theorem.
18. a) Let X and Y be normed linear spaces and $A: X \rightarrow Y$ is an linear operator, Prove that the following are equivalent:
i) A is continuous at 0
ii) A is continuous at every $x \in X$.
iii) A is uniformly continuous
iv) There exists $c > 0$ such that $\|Ax\| \leq c\|x\|$ for all $x \in X$
v) $\{Ax : x \in X, \|x\| = 1\}$ is the bounded subset of Y .
(OR)
b) State and Prove Bessel's Inequality.
19. a) State & prove Hahn Banach extension Theorem.
(OR)
b) State and Prove Separation Theorem.
20. a) State and Prove Arzela-Ascoli Theorem.
(OR)
b) Let X and Y be Banach spaces. Prove that every closed linear operator $A: X \rightarrow Y$ is continuous.