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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS311

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

COURSE NAME: M.Sc.- MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: III **TIME : 3 HOURS**

TOPOLOGY

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)
MULTIPLE CHOICE QUESTIONS.

1. Every finite point set in Hausdorff space X is ________.
   a) finite b) closed c) connected d) countable
2. If A is bounded and nonempty then the diameter of A is _______.
   a) inf $\{d(x,y),1\}$ b) sup$\{d(x,y),1\}$
   c) sup$\{d(a_1,a_2) \mid a_1,a_2 \in A\}$ d) inf $\{d(a_1,a_2) \mid a_1,a_2 \in A\}$
3. If $f: X \to Y$ be a bijective continuous function, where X is compact and Y is Hausdorff, then f is a ______.
   a) Connected b) Compact c) Homeomorphism d) Homomorphism
4. A subset D of a topological space X is said to be dense in X if _______.
   a) $\overline{D} = X$ b) $D = X$ c) $X \subset \overline{D}$ d) $D \neq X$
5. A Compactification of a space X is a compact Hausdorff space Y containing X as a Subspace such that _______.
   a) $\overline{X} = Y$ b) $X = Y$ c) $\overline{X} \neq Y$ d) $X \neq Y$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define Subspace Topology.
7. Define Uniform Topology.
8. Define locally connected.
9. State Urysohn lemma.
10. State Nagata-Smirnov metrization theorem.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that the Topologies of $\mathbb{R}_l$ *and* $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$.

(OR)

b) Prove that a subspace of a Hausdorff Space is a Hausdorff Space.

12. a) State and Prove pasting lemma.

(OR)

b) State and Prove the sequence lemma.

(CONTD……2)

13.a) Prove that the image of a connected space under a continuous map is connected.

(OR)

b) Prove that an every compact subspace of a Hausdorff space is closed.

14.a) Prove that an every metrizable space is normal.

(OR)

b) Prove that a product of completely regular spaces is completely regular.

15.a) If X is completely regular space then show that Stone-Cech compactification exists.

(OR)

b) Let X be normal and A be a closed G_δ set in X. Prove that there is a continuous function $f: X \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Let X be a topological space, Prove that the following conditions hold,
i) ϕ and X are closed
ii) Arbitrary intersections of closed sets are closed
iii) Finite union of closed sets are closed.

(OR)

b) Prove that if Y be a subspace of X then a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

17. a) Let the functions f_1:A→X and f_2:A→Y be defined on A. Let f: X x Y be defined by the equation $f(a) = (f_1(a), f_2(a))$, prove that f is continuous if and only if the functions f_1 and f_2 are continuous.

(OR)

b) Let $f: X \to Y$. If the function f is continuous then prove that for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$. Prove also that the converse holds if $X$is metrizable.

18. a) Prove that if L is a linear continuum in the order topology, then L is connected, and so are intervals and rays in L.

(OR)

b) Prove that the product of finitely many compact spaces is compact.

19. a) Prove that an every regular space with a countable basis is normal.

(OR)

b) State and Prove Tietze's Extension Theorem.

20. a) State and Prove Tychonoff Theorem.

(OR)

b) State and Prove Smirnov metrization Theorem.

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