(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2023 ONLY)

23UMS102

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

COURSE NAME: B.SC(MATHEMATICS) **MAXIMUM MARKS: 75**

SEMESTER: I **TIME : 3 HOURS**

PART - III

CALCULUS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
(OBJECTIVE QUESTIONS WITH FOUR MULTIPLE CHOICES) (K1)

1. The reciprocal of the curvature of a curve at any point is called _______.
   (a) Center of curvature (b) radius of curvature (c) evolute (d) envelope
2. Elimination of a and b from $z = (x+a)(y+b)$ __________.
   (a) $z = ab$ (b) $z = a + b$ (c) $z = p + q$ (d) $z = pq$
3. The value of $\int_0^a \int_0^a \int_0^a dx\ dy\ dz$ is________.
   (a) a^3 (b) a^2 (c) a (d) 1
4. $L[e^{-at}] =$ ____ .
   (a) $\frac{1}{s+a}$ (b) $\frac{1}{s-a}$ (c) $s+a$ (d) $\frac{1}{s}$
5. $n! =$______
   (a) $\Gamma(1)$ (b) $\Gamma(n)$ (c) $n\Gamma(n)$ (d) n

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Solve $(D^2 + 5D + 4)y = 0$ *where* $D = \frac{d}{dx}$
7. Equations, in which the variables do not occur explicitly, can be written in the form
8. Write double integral in polar coordinates
9. What is called Jacobian?
10. Find $\Gamma\left(\frac{1}{2}\right)$

SECTION – B (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Find the envelope of the family of straight lines $y + tx = 2at + at^3$, the parameter being t

(OR)

b) Solve $(D^4 + 4)y = x \sin x$.

12. a) Form a partial differential equation by eliminating the arbitrary function f from $f(x^2+y^2+z^2, z^2-2xy)$.

(OR)

b) Interpret the solution of the equation $p^2+q^2=npq$.

13. a) Calculate $\iint xy\,dx\,dy$ taken over the positive quadrant of the circle $x^2+y^2=a^2$.

(OR)

b) By changing the order of integration , Evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dxdy$.

14. a) Prove that $\beta(m,n)=\beta(m)\beta(n)$.

(OR)

14 b) Evaluate $\iint_R (x-y)^2 e^{x+y}\,dx\,dy$ where R is the square with vertices (1,0),(2,1), (2,1) and (0,1)

15.a) Show that $L[\frac{\cos 3t-\cos 2t}{t}]=\frac{1}{2}\log(\frac{s^2+4}{s^2+9})$

(OR)

15 b) Prove that $\int_0^\infty \frac{e^{-t}-e^{-2t}}{t}dt=\log 2$

SECTION – C (5 X 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Prove that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$ is $4a \cos\frac{\theta}{2}$

(OR)

b) Solve $(D^3+1)y=x^2e^{2x}+x\cos x$.

17.a) Solve the equation $px(y^2+z)-qy(x^2+z)=z(x^2-y^2)$ Also find the surface that contains the straight line $x+y=0, z=1$

(OR)

b) Solve $(y+z)p+(z+x)q=x+y$.

18.a) Justify $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$, by transforming into spherical co-ordinates.

(OR)

b) Find the centroid of a loop of the lemniscates $r^2=a^2\cos 2\theta$

19.a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(OR)

b) Use the substitution $x + y + z = u, y + z = uv, z = uvw$ to evaluate the integral $\int\int\int [xyz(1 - x - y - z)]^{1/2}\, dxdydz$ taken over the tetrahedral volume enclosed by the planes $x = 0, y = 0, z = 0$ *and* $x + y + z = 1$

20.a) Using Laplace transform, Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = \sin t$, given that $y = \frac{dy}{dt} = 0$ where t=0

(OR)

b) Show that the solution of the differential equation $\frac{d^2y}{dx^2} = 4y = A\sin At$ which is such that $y = 0$ *and* $\frac{dy}{dt} = 0$ *when* $t = 0$ is $y = A\frac{\sin At - \frac{k}{2}\sin 2t}{4 - k^2}$ *if* $k \neq 2$. If $k = 2$ show that $y = \frac{A(\sin 2t - 2t \sin 2t}{6}$

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ETHICAL PAPER