(FOR THE CANDIDATES ADMITTED
DURING THE ACADEMIC YEAR 2023 ONLY)

23UMS101

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS: NOVEMBER 2023

COURSE NAME : B.Sc.- MATHEMATICS   **MAXIMUM MARKS: 75**

SEMESTER: I   **TIME: 3 HOURS**

PART - III
CLASSICAL ALGEBRA

SECTION – A   **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS.   **(K1)**

1. _____ method for obtaining approximate solutions for algebraic equations.

   (a) Bisection method   (b) Newton- Raphson's method   (c) Regula Falsi method   (d) all

2. $(1 + x)^n = $ _____

   (a) $1 - nx + \dfrac{n(n-1)}{2!}x^2 - \dfrac{n(n-1)(n-2)}{3!}x^3 + \dots\dots$.

   (b) $1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \dots\dots$.

   (c) $1 - nx + \dfrac{n(n+1)}{2!}x^2 - \dfrac{n(n+1)(n+2)}{3!}x^3 + \dots\dots$.

   (d) $1 + nx + \dfrac{n(n+1)}{2!}x^2 + \dfrac{n(n+1)(n+2)}{3!}x^3 + \dots\dots$.

3. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ then $\sum \alpha = $ ____

   (a) $-p$ (b) q (c) $-r$ (d) $3r - pq$

4. A real fraction cannot be a root of an equation with integral coefficients, the coefficient of x^n being_____

   (a) ∞ (b) 0 (c) $-\infty$ (d) 1

5. A square matrix A is said to be unitary if_____.

   (a) $AA^T = I$ (b) $A = A^T$ (c) $A\overline{A}^T = \overline{A}^T A = I$ (d) $A\overline{A} = I$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.   **(K2)**

6. Write the formula of Newton- Raphson's method.

7. Write the formula of $(1 - x)^{p/q}$

8. What is called symmetric function of the roots?

9. State the Strum's theorem.

10. Write the statement of Cayley Hamiltonian Theorem.

SECTION – B (5 X 8= 40 MARKS)
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11(a). Perform four iterations of the Newton-Raphson's method to find the smallest positive root of the equation $f(x) = \cos x - xe^x$.

(OR)

(b). A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval $(0,1)$, perform four iteration of Regula Falsi method to find the root.

12(a) Determine the coefficient of x^n in the expansion of $\dfrac{1 + 2x - 3x^2}{e^x}$

(OR)

(b) Sum to n terms the series $3.5.7 + 5.7.9 + 7.9.11 + ...$

13. a) If $a + b + c + d = 0$ show that $\dfrac{a^5 + b^5 + c^5 + d^5}{5} = \dfrac{a^2 + b^2 + c^2 + d^2}{2} \cdot \dfrac{a^3 + b^3 + c^3 + d^3}{3}$

(OR)

b) Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

14 (a Find the multiple roots of the equation $x^4 - 9x^2 + 4x + 12 = 0$.

(OR)

(b) Find the rational root of $2x^2 - x - 3 = 0$.

15.a) Show that a square matrix A is orthogonal iff $A^{-1} = A^T$

(OR)

b) Find the characteristic roots of the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K4 (Or) K5)

16. a) Find a real root of the equation $x^3 - 3x + 1 = 0$ lying between 1 and 2 correct to three places of decimal by using bisection method.

(OR)

b) Solve $f(x) = 3x + \sin x - e^x$ by using Newton's method, take the point $x_0 = 0$

17. a) Evaluate the sum to infinity of the series

$$\frac{15}{16} - \frac{15.21}{16.24} + \frac{15.21.27}{16.24.32} - ....$$

(OR)

17 b) (i) Show that $\log \sqrt{12} = 1 + \left(\dfrac{1}{2} + \dfrac{1}{3}\right)\dfrac{1}{4} + \left(\dfrac{1}{4} + \dfrac{1}{5}\right)\dfrac{1}{4^2} + \left(\dfrac{1}{6} + \dfrac{1}{7}\right)\dfrac{1}{4^3} + ...$

18. a) If $\alpha, \beta, \gamma, \delta$ are the roots of biquadratic equation $x^3 + ax^2 + bx + c = 0$ form the equation whose roots are $\beta + \gamma - 2\alpha$, $\gamma + \alpha - 2\beta$, $\alpha + \beta - 2\gamma$.

(OR)

b) Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.

19.a) Evaluate the roots of the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$ by finding the rational roots.

(OR)

b) The equation $x^3 - 3x + 1 = 0$ has a root between 1 and 2 . Calculate it to three places of decimals.

20.a) Let A be a square matrix then prove that

(i) The sum of the Eigen values of A is equal to the sum of the diagonal elements(trace) of A

(ii) Product of Eigen values of A is $|A|$

(OR)

20 b) Find the Eigen values and Eigen vectors of the matrix A=$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

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ETHICAL PAPER