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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2021 ONLY)

21UMS512

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: V **TIME : 3 HOURS**

PART - III

THEORY OF NUMBERS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. What is the formula to compute the sum of the first *n* positive integers?

   a) $\frac{(n-1)}{2}$ b) $\frac{n(n+1)}{2}$ c) $\frac{n(n-1)}{2}$ d) $\frac{n}{2}$

2. If h and j are two integers and $h \equiv j \pmod{m}$, then how j is called with respect to h modulo m?

   a) congruence b) complete residue system c) perfect square d) residue

3. Which of the following conditions is the necessary and sufficient condition for the congruence $(m-1)! \equiv -1 \pmod{m}$ to hold?

   a) m is a prime b) m is composite c) m is zero d) m is odd prime

4. What is the condition for a function f(mn) = f(m) f(n) to be multiplicative?

   a) $m|n$ b) gcd (m, n) = 1 c) n|m d) lcm (m, n) = 1

5. What is the value of $\lim_{x\to\infty} \frac{\pi(x)}{x}$?

   a) 0 b) 1 c) $-\infty$ d) $+\infty$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Explain the conditions for an integer d to be the greatest common divisor of another integer a.
7. Define Euler function with an example.
8. Provide a suitable illustration to the statement " If gcd (a, c) = 1, then a has an inverse and it is unique modulo c".
9. Define the Möbius function with an Example.
10. Define primitive root modulo m with an example.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Sketch the binary notations for 47, 68 and 127

(OR)

   b) If a, b and c are integers where a and c are relatively prime and if c | ab then show that c | b

(CONTD…..2)

12.a) If ${}_nP_r$ denotes the number of r-permutations of a set of n objects, then show that

${}_nP_r = n\,(n-1)\ldots(n-r+1)$

(OR)

b) If s different integers $r_1, r_2, \ldots, r_s$ form a complete residue system modulo m, then show that s = m.

13.a) State and prove Fermat's little theorem

(OR)

b) Find the least positive integer x such that the congruences $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$ and $x \equiv 2 \pmod 7$ holds.

14.a) Compute the values of $\varphi(19)$, $\varphi(49)$, $\varphi(243)$ and $\varphi(1024)$

(OR)

b) Show that

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1 \end{cases}$$

15.a) Show that there are no primitive roots modulo 8

(OR)

b) If [x] denotes the largest integer that does not exceed x, then show that

$$0 \le [2x] - 2[x] \le 1$$

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS
(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Construct the general formulae for d(n) and σ(n)
17. State and prove the basis representation theorem
18. Prove that $a_n = \binom{2n-2}{n-1}/n$.
19. (i) State the Chinese Remainder Theorem.

    (ii) Evaluate the congruence $3x \equiv 11 \pmod{2275}$ using the Chinese Remainder Theorem.
20. Derive the Mobius Inversion Formula
21. Prove that for each prime p, there exist primitive roots modulo p.

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