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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2021 ONLY)

21UMS509

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : NOVEMBER-2023

COURSE NAME: B. Sc.- MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: V **TIME : 3 HOURS**

PART - III

MODERN ALGEBRA

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. Two sets are said to be disjoint if their ______ is empty.
   a) union b) intersection c) complement d) null set
2. $\phi(x) = \bar{e}$, $\bar{e}$ is ______ element of $\bar{G}$.
   a) product b) nilpotent c) kernel d) identity
3. Every permutation can be uniquely expressed as a product of disjoint______ .
   a) paths b) lines c) cycles d) sum
4. A ring with this latter property is called a ______.
   a) field b) elements c) divisor d) finite
5. In the Euclidean ring R, a and b in R are said to be relatively prime if their ______ common divisor is a unit of R.
   a) lowest b) upper c) greatest d) bounded

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define abelian group.
7. Write the equation of normal subgroup.
8. What is normalizer?
9. Define Homomorphisms.
10. Write two points of greatest common divisor.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If $\sigma : S \to T$, $\tau : T \to U$ and $\mu : U \to V$, then $(\sigma \circ \tau) \circ \mu = \sigma \circ (\tau \circ \mu)$.

(OR)

b) If H is a non empty finite subset of a group G and H is closed under multiplication, then H is a subgroup of G.

(CONTD…..2)

12.a) Prove that N is a normal subgroup of G if and only if gNg^{-1}=N for every $g \in G$.

(OR)

b) Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K, and let $\overline{N}$ be a normal subgroup of $\overline{G}$,

$N = \{x \in G / \phi(x) \in \overline{N}\}$. Then $G/N \cong \overline{G}/\overline{N}$ equivalently, $G/N \approx (G/K)/(N/K)$.

13.a) Every permutation is the product of its cycles.

(OR)

b) N(a) is a subgroup of G.

14.a) If R is a ring, then $\forall\, a, b \in R$ prove that 1. A0=0a=0 2. a(-b) = (-a)b = -(ab).

(OR)

b) Prove that a finite integral domain is a field.

15.a) Prove that Euclidean ring possesses a unit element.

(OR)

b) If p is a prime number of the form 4n+1, then prove that $p=a^2+b^2$ for some integers a, b.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16^{th} QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS
(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. If p is prime number and p/o(G), then prove that G has an element of order p.

17. If H and K are finite subgroups of G of orders o(H) and o(K) respectively, then prove that

$$o(HK) = \frac{o(H)o(K)}{o(H \cap K)}.$$

18. If ϕ is a homomorphism of G onto $\overline{G}$ with kernel K, then prove that $G/K \approx \overline{G}$.

19. Prove that every group is isomorphic to a subgroup of A(S) for some appropriate S. Define automorphism and conjugate.

20. Prove that finite integral domain is a field.

21. Prove that J[i] is a Euclidean ring.

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ETHICAL PAPER