(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2019 ONLY)

19UMS613 / 19UMA613

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : JULY-2022

B.Sc.-MATHEMATICS **MAXIMUM MARKS: 75**

SEMESTER : VI **TIME : 3 HOURS**

PART - III

LINEAR ALGEBRA

SECTION – A **(10 X 1 = 10 MARKS)**

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. If A x B are m x n matrices over the field F, then B is __________to A if B can be obtained from A by a finite sequence of elementary row operations.
   a) column-equivalent b) row-equivalent c) subfield d) row-reduced

2. If W is a proper subspace of a finite-dimensional vector space V, then W is finite-dimensional and _________.
   a) dim W < dim V b) dim W > dim V c) dim W = dim V d) dim W = 0

3. A group is called ____________ if it satisfies the condition xy = yx for each x and y.
   a) invertible b) Associative c) isomorphic d) commutative

4. If A is an n x n matrix with entries in field F, then tr A = _________.
   a) $A_{11}+A_{12}+….+ A_{1n}$ b) $A_{11}+A_{21}+….+ A_{n1}$ c) $A_{11}+A_{22}+….+ A_{nn}$ d) 0

5. If V is a vector space, then a ___________ in V is a maximal proper subspace of V.
   a) hyperspace b) linear subspace c) functional d) null space

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**

6. When do you say an m x n matrix is row-reduced?
7. Define linearly dependent of a subset.
8. Define linear transformation.
9. When do you say the matrix B is similar to A with dimension n x n?
10. Define the transpose of an n x m matrix.

SECTION – B **(5 X 5 = 25 MARKS)**

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If A is an n x n matrix, prove that A is row-equivalent to the n x n identity matrix iff the system of equations AX = 0 has only the trivial solution.

(OR)

b) Compute ABC and CAB if $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \end{bmatrix}$.

(CONTD……..2)

12.a) Prove that a non-empty subset W of V is a subspace of V iff for each pair of vectors α, β in W and each scalar c in F the vector $c\alpha + \beta$ is again in W.

(OR)

b) If R be a non-zero row-reduced echelon matrix, prove that the non-zero row vectors of R form a basis for the row space of R.

13. a) If V and W be vector spaces over the field F and let T be a linear transformation from V into W. If T is invertible, prove that the inverse function T^{-1} is a linear transformation from W into V.

(OR)

b) If A is an m xn matrix with entries in the field F, then prove that row rank (A) = column rank (A).

14. a) If V be a finite-dimensional vector space over the field F, and let $\mathfrak{B}^* = \{\alpha_1, \alpha_2, \dots \alpha_n\}$ be a basis for V, prove that there is a unique dual basis $\mathfrak{B}^* = \{f_1, f_2, \dots f_n\}$ for V* such that $f_i(\alpha_j) = \delta_{ij}$. For each linear functional f on V we have $f = \sum_{i=1}^{n} f(\alpha_i) f_i$ and for each vector α we have $\alpha = \sum_{i=1}^{n} f_i(\alpha)\alpha_i$.

(OR)

b) If V be the space of all polynomial function from R into R of the form $f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$, find the matrix D in the ordered basis $\mathfrak{B}$.

15. a) If S is any subset of a finite-dimensional vector space V, prove that $(S_0)^0$ is the subspace spanned by S.

(OR)

b) If g, $f_1, \dots f_r$ be linear functionals on a vector space V with respective null spaces N, $N_1, \dots N_r$, prove that g is a linear combination of $f_1, \dots f_r$ iff N contains the intersection $N_1 \cap \dots \cap N_r$.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS

(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. If V be a vector space which is spanned by a finite ste of vector $\beta_1, \beta_2, \dots \beta_m$, prove that any independent set of vectors in V is finite and contains no more than m elements. **(K4)**

17. Prove that for an n x n matrix A, the following are equivalent (i) A is invertible (ii) The homogeneous system AX = 0 has only the trivial solution X = 0.(iii) The system of equation AX = Y has a solution X for each n x 1 matrix Y. **(K4)**

18. If W_1 and W_2 are finite-dimensional subspaces of a vector space V, prove that W_1+W_2 is finite-dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$. **(K5)**

19. Let V be a finite-dimensional vector space over the field F and let $\{\alpha_1, \dots, \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same filed F and let $\beta_1, \dots, \beta_n$ be any vectors in W. Prove that there is precisely one linear transformation T from V into W such that $T_{\alpha j} = \beta_j$, j = 1, …, n. **(K4)**

20. If W be the subspace of R^5 which is spanned by the vectors $\alpha_1 = (2, -2, 3, 4, -1)$, $\alpha_2 = (-1, 1, 2, 5, 2)$, $\alpha_3 = (0, 0, -1, -2, 3)$, $\alpha_4 = (1, -1, 2, 3, 0)$, describe W^0, the annihilator of W. **(K4)**

21. Let V and W be finite-dimensional vector spaces over the field F. Let $\mathfrak{B}$ be an ordered basis for V with dual basis $\mathfrak{B}^*$, and let $\mathfrak{B}'$ be an ordered basis for W with dual basis $\mathfrak{B}'^*$. Let T be a linear transformation from V into W, let A be the matrix of T relative to $\mathfrak{B}$, $\mathfrak{B}'$ and let B be the matrix of T^t relative to $\mathfrak{B}'^*$, $\mathfrak{B}^*$. Prove that $B_{ij} = A_{ji}$. **(K5)**

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