(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2020 ONLY)

20PMS4E5

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : JULY- 2022

M.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: IV **TIME : 3 HOURS**

MATHEMATICAL METHODS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. If g(s) = $\lambda \int_0^{\pi} (sin\, s \ \sin 2t) g(t) dt$ has ______ eigen values.
   a) one b) two c) three d) no
2. Initial value problem is reduced to ______ integral equation with given initial conditions.
   a) Fredholm b) Volterra c) both d) Abel's
3. A functional V[y(x)] reaches a maximum on a curve $y = y_0(x)$,with the difference $y(x) - y_0(x)$ as small, then the maximum is called ________.
   a) strong b) weak c) strictly maximum d) strictly minimum
4. If the extremal C is included in the field of extremals then the condition can be replaced by ___
   a) Legendre condition b)Weiestrass condition c) Jacobi condition d) minimum condition
5. The equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f(x, y)$ is known as ______.
   a) Laplace equation b) Poisson equation c) Euler equation d) Jacobi equation

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES **(K2)**

6. Write the Neumann series.
7. Write the Fredholm integral equation.
8. What is Ostrogradsky equation?
9. Define Weiestrass function.
10. Write the approximate solution of the Kantorovich's method.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. **(K3)**

11. a) Solve the volterra integral equation g(s) = 1 $+ \int_0^s st\ g(t) dt$.

(OR)

b) Find the resolvent kernel for the integral equation g(s) = f(s) $+\lambda \int_{-1}^{1} (st + s^2 t^2)\ g(t) dt$

12.a) Reduce the Initial value problem y"(s)+ λy(s) = F(s) with the initial conditions y(0) = 1,y'(0) = 0 to a volterra integral equation.

(OR)

b) Solve the Abel's Integral equation f(s) = $\int_0^s \frac{g(t)}{(s-t)^{\alpha}}$ dt ,0 < α < 1.

(CONTD……2)

13. a) State and prove the fundamental theorem of calculus of variation.

(OR)

b) Find the extremals for the functional $V[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y_1^2}}{x} dx$.

14. a) Is the Jacobi condition fulfilled for the extremal of the functional $v[y(x)] = \int_0^a (y'^2 + y^2 + x^2) dx$, that passes through the points A(0,0) and B(a,0)?

(OR)

b) Test for an extremum for the functional

$$v = \int_0^a y'^3 dx, \quad y(0) = 0, y(a) = b, a > 0, b > 0$$

15. a) Find an approximate solution associated with the torsion of a cylinder or prism, to investigate the functional for an extremum $v[z((x,y)] = \iint [(\frac{\partial z}{\partial x} - y)^2 + (\frac{\partial z}{\partial y} + x)^2] dx dy.$

(OR)

b) Find a continuous solution of the equation $\Delta z = -1$ in the domain D, which is an isosceles triangle bounded by the straight lines $y = \pm \frac{\sqrt{3}}{3} x$ and $x = b$ and the solution vanishes on the boundary of the domain.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS

(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS

(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Solve the integral equation g(s) = f(s) $+\lambda \int_0^1 (1 - 3st) g(t) dt.$

17. Solve the integral equation g(s) = (e^s –s) - $\int_0^1 s(e^{st-1}) g(t) dt$ using approximation method.

18. From a boundary value problem y''(s)+ A(s)y'(s)+B(s) y(s) = F(s) with y(a) = y_0, y(b) = y_1 reduce a Fredholm type integral equation y(s) = f(s)+ $\int k(s,t) y(t) dt$.

19. State and Prove Brachistrone problem.

20. Find the equation of geodesics on a surface on which the element of length of the curve is of the form $ds^2 = \{\varphi_1(x) + \varphi_2(y)\}(dx^2 + dy^2).$

21. Using the Ritz method, find an approximate solution of the differential equation $y'' + x^2 y = x; y(0) = y(1) = 0.$ Determine $y_2(x)$ *and* $y_3(x)$ and compare their values at the points $x = 0.25, x = 0.5$ *and* $x = 0.75.$

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