(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2021 ONLY)

21PMS207

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : JULY- 2022

M.Sc -MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER : II **TIME : 3 HOURS**

PARTIAL DIFFERENTIAL EQUATIONS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. **(K1)**

1. Find the differential equation of all spheres whose centers lie on the z-axis__________.
   (a) xq = yp (b) xp = yq (c) x+p = y+q (d) x/p = y/p
2. If $u_1, u_2, \ldots u_n$, are solutions of the homogenous linear partial differential equation F(D,D')z = 0, then___________.
   (a) $\sum_{r=0}^{n} c_r u_r$ **(b)** $\sum_{r=1}^{n} c_r u_r$ (c) $\sum_{r=1}^{n} c_k k$ (d) $\sum_{r=0}^{n} u_r$
3. The solution of the partial differential one – dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ is_____.
   (a) $Z(x,t) = C_n \sin(nx+\varepsilon_n)e^{-n^2 kt}$ (b) $Z(x,t) = C_n \sin(nx+\varepsilon_n)e^{n^2 kt}$
   (c) $Z(x,t) = C_n \cos(nx+\varepsilon_n)e^{-n^2 kt}$ (d) $Z(x,t) = C_n \sin(nx - \varepsilon_n)e^{n^2 kt}$
4. The surfaces f(x,y,z) = c will be equipotential if the potential function ψ is _________ whenever f(x,y,z) is a constant.
   (a) constant (b) zero (c) equal (d) unequal
5. Assuming the time and space variables can be separated , the equation $\nabla^2\theta = \frac{1}{k}\frac{\partial \theta}{\partial t}$ has the solution of the form___________.
   (a) θ ≠ φ(r)T(t) (b) θ = φ(r)T(t) (c) θ = φ(r) + T(t) (d) θ = φ(r) - T(t)

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Define complete integral and particular integral.
7. Classify the linear differential operators F(D, D') into two main types.
8. Write the procedure to be followed in applying the theory of integral transforms to the solution of partial differential equations.
9. Define the two main boundary value problems for Laplace's equation.
10. Write the d'Alembert's solution of the one-dimesional wave equation.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Find a complete Integral of the equation $p^2y(1+x^2) = qx^2$

(OR)

b) Show that the equations xp = yq , z(xp + yq) = 2xy are compatible and solve them.

12.a) Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$ is satisfied by $z = \frac{1}{x}\,\phi(y-x)+\phi'(y-x)$ where ϕ is an arbitrary function.

(OR)

b) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^2} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$

13.a) Derive the solution of the equation: $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$, for the region $r \geq 0$, $z \geq 0$, satisfying the conditions (i) $V \to 0$ as $z \to \infty$ and $r \to \infty$ (ii) $V = f(r)$ on $z = 0$, $r \geq 0$.

(OR)

b) Solve the equation $q^2 r - 2pqs + p^2 t = 0$.

14.a) Show that the surfaces $x^2 + y^2 + z^2 = \mathbf{c}x^{2/3}$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.

(OR)

b) A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

15.a) The points of trisection of a string are pulled aside through a distance a on the opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

(OR)

b) Find approximate values for the first three eigen values of square membrane of side 2.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) **(K4 (Or) K5)**

16. A uniform circular wire of radius a charged with electricity of line density e surrounds grounded concentric spherical conductor of radius c. Determine the electric charge density at any point on the conductor.

17. Solve by Jacobi's method: $p^2 x + q^2 y = z$

18. Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

19. Determine the solution of the equation: $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0$ $(-\infty < x < \infty,\ y \geq 0)$ satisfying the conditions (i) z and its partial derivatives tend to zero as $x \to \pm\infty$; $z = f(x)$, $\frac{\partial z}{\partial y} = 0$ on $y = 0$.

20. A uniform insulated sphere of dielectric constant k and radius a carries on its surface a charge of density $\lambda P_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount $\frac{8\pi^2\lambda^2 a^3 \kappa n}{(2n+1)(\kappa n + n + 1)^2}$ to the electrostatic energy.

21. A thin membrane of great extent is released from rest in the position $z = f(x,y)$. Determine the displacement at any subsequent time.

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