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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2021 ONLY)

21PMS206

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS: JULY- 2022

M.Sc.- MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: II **TIME : 3 HOURS**

MATHEMATICAL STATISTICS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. (K1)

1. Suppose the random variable X can take on two values x_1 = -1 with probability p_1 =0.1 and x_2 = +1 with probability p_2 = 0.9, then E(X) =____________
   (a)0.2 (b)0.8 (c)0.5 (d)0.6
2. The characteristic function of the random variable X is given by $\phi(t) = \exp(\frac{-t^2}{2})$.The density function of this random variable is _____________
   (a) $\frac{1}{\sqrt{2\pi}}\exp(\frac{x}{2})$ (b) $\frac{1}{\sqrt{2}}\exp(\frac{-x^2}{2})$ (c) $\frac{1}{\sqrt{2\pi}}\exp(\frac{-x^2}{2})$ (d) $\frac{1}{\sqrt{\pi}}\exp(\frac{-x^2}{2})$
3. Γ (p+1) =____________
   (a)p (b)Γ(p) (c)Γ(p+1) (d)pΓ(p)
4. Let the independent random variables X_k (k = 1,…n)have the same normal normal density $f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{1}{2}\frac{x^2}{\sigma^2})$. Then the statistic χ^2= ____________.
   (a) $\sum_{k=1}^{n-1} X_k^2$ (b) $\sum_{k=1}^{n} X_k^2$ (c) $\sum_{k=0}^{n-1} X_k^2$ (d) $\sum_{k=0}^{n} X_k^2$
5. The Unbiased Estimate U = $\frac{n}{n-1}S^2$, Its efficiency is
   (a) e < 1 (b)e > 1 (c)e = 1 (d) e = 0

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Compute the probability that head appears atleast twice in successive tosses of a coin.
7. Prove that the characteristic function of the sum of an arbitrary finite number of independent random variables equals the product of their characteristic functions.
8. When is the sequence $\{X_n\}$ stochastically convergent to zero?
9. Define statistical hypothesis with an example.
10. When do you say U_n of estimates is consistent? Give an example.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If a random variable Y can take on only non-negative values and has expected value E(Y), prove that for an arbitrary positive number k , $P(Y \geq k) \leq \frac{E(Y)}{k}$.

(OR)

b) Gun 1 and 2 are shooting at the same time. It has been found that gun 1 shoots on the average 9 shots during the same time gun 2 shoots 10 shots. The precision of these two guns is not the same, on the average, out of 10 shots from gun 1, eight hit the target, and from gun 2, only seven. During the shooting the target has been hit by a bullet, but it is not known which gunshot this bullet. What is the probability that the target was hit by gun 2?

12.a) If the moment m_l of a random variable exists, prove that $m_l = \frac{\phi^{(l)}(0)}{i^l}$, where $\phi^{(l)}(0)$ is the lth derivative of the characteristic function $\phi(t)$ of this random variable at t = 0.

(OR)

b) Let the random variable X_n have a binomial distribution defined by the formula $P(X_n = r) = \frac{n!}{r!(n-r)!} p^r (1-p)^{n-r}$, where r takes the values 0,1,2,…n. If for n = 1,2… the relation $p = \frac{\lambda}{n}$ holds , where $\lambda > 0$ is a constant , prove that $\lim_{n\to\infty} P(X_n = r) = \frac{\lambda^r}{r!} e^{-\lambda}$

13.a) The random variable X has the distribution N(1;2). Find the probability that X is greater than 3 in absolute value.

(OR)

b) Prove that the sequence of random variables $\{X_n\}$where $P(Y_n = \frac{r}{n}) = \binom{n}{r} p^r (1-p)^{n-r}$ where $0 < p < 1$and r take values 0,1,2….n. and $X_n = Y_n - P$ is stochastically convergent to 0,

14.a) There are good and defective items in a lot and the proportion p of defective items is unknown. The hypothesis to be tested is H_0(p = 0.10) and hence verify that 10% of the items in the lot are defective.

(OR)

b) Suppose in a sample of $n = 150$ elements, the mean and standard deviation of the sample are $\bar{x} = 0.4$ and s= 4. Should the hypothesis H_0 be rejected or not?

15.a) Let the distribution function F(x)depend upon one parameter Q, that is m = 1.If there exists a sufficient estimate U of the parameter Q, prove that the solution of the equation $\frac{\partial logL}{\partial \lambda_i} = 0$ (i = 1,3,….,m) is a function of U only

(OR)

b) Let V be an unbiased estimate of the parameter Q and let U be a sufficient estimate of Q. Prove that the random variable E(V/u) is an unbiased estimate of Q. If the variance exists, the inequality $D^2[E(V/u)] \le D^2(V)$ holds only if E(V/u) = V with probability one.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS

(16^{th} QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. State and Prove De Moivre-Laplace Theorem.
17. State and Prove the Lapunov Inequality for a random variable.
18. Compute the characteristic function and moments of a normal distribution.
19. A box contains a collection of IBM cards corresponding to the workers from some branch of industry. Of the workers 20% are minors and 80% adults. We select one IBM card in a random way and mark the age given on this card. Before choosing the next card, we return the first one to the box, so that the probability of selecting the card corresponding to a minor remains 0.2. We observe n cards in this manner. What value should n have in order that the probability will be 0.95 that the frequency of cards corresponding to minor lies between 0.18 and 0.22?
20. Compute the density function for the student's t -ststistic.
21. Prove that an unbiased estimate U of the parameter Q is the most efficient if and only if

    (i) the estimate U is sufficient

    (ii) for $g(u,Q) > 0$, the density $g(u,Q)$ almost everywhere satisfies the relation .

$$\frac{\partial \log g(u,Q)}{\partial Q} = c(u-Q)$$

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