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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS208

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY-2023

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: II **TIME : 3 HOURS**

MECHANICS

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

MULTIPLE CHOICE QUESTIONS.

1. D'Alembert's principle states that the sum of all forces, real and inertial, acting on each particle of a system is ____.
   (a) Scleronomic (b) conservative (c) infinity (d) zero
2. The standard form of Lagrange's equation for a non holonomic system can also be applied to a holonomic system in which there are more generalized coordinates than _____.
   (a) Routhian function (b) degrees of freedom
   (c) angular momentum (d) ignorable coordinates
3. The necessary and sufficient condition that f have a stationary value at q_0 is that ____ for all geometrically possible δq's where $q = q_0 + \delta q$.
   (a) $f = 0$ (b) $f = c$ (c) $\delta f = 0$ (d) $\delta f = c$
4. The function $s(q_0, q_1, t_0, t_1)$ is assumed to be twice differentiable in all its arguments and is known as ______ function.
   (a) Jacobi (b) holonomic (c) Hamilton's principle (d) continious
5. A general characteristic of the Lagranges bracket is that its value is ____ under a canonical transformation.
   (a) invariant (b) constant (c) zero (d) Poisson bracket

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. State the principle of virtual work.
7. Write the standard form of Lagrange's equation for a holonomic system.
8. Write the Hamilton principle.
9. State Jacobi's theorem.
10. State Poisson's theorem.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Convert Cartesian to generalized coordinates of a particle which is constrained to move on a fixed path of radius 'a' whose equation is $\left(x_1^2 + x_2^2\right)^{\frac{1}{2}} = a$.

(OR)

b) Explain the conservation of energy.

(CONTD……2)

12.a) Find the differential equations of motion for a spherical pendulum of length *l*.

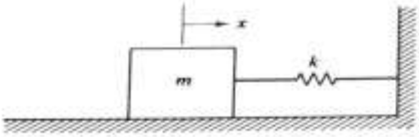
(OR)

b) Explain natural systems.

13.a) Find the stationary values of the function $f = z$, subject to the constraints $\phi_1 = x^2 + y^2 + z^2 - 4 = 0$, $\phi_2 = xy - 1 = 0$.

(OR)

b) Given a mass-spring system consisting of a mass m and a linear spring of stiffness k, as shown in below. Find the equations of motion using the Hamiltonian procedure. Assume that the displacement x is measured from the unstressed position of the spring.



14. a) Describe Hamilton's principal function.

(OR)

b) Describe Pfaffian differential forms.

15. a) Consider the transformation $Q = \frac{1}{2}\left(q^2 + p^2\right)$, $P = -\tan^{-1}\frac{q}{p}$ show that this transformation is canonical.

(OR)

b) Consider the transformation $Q = \sqrt{e^{-2q} - p^2}$, $P = \cos^{-1}\left(pe^q\right)$. Use the Poisson bracket to show that it is canonical.

SECTION – C (5 X 5 = 25 MARKS)

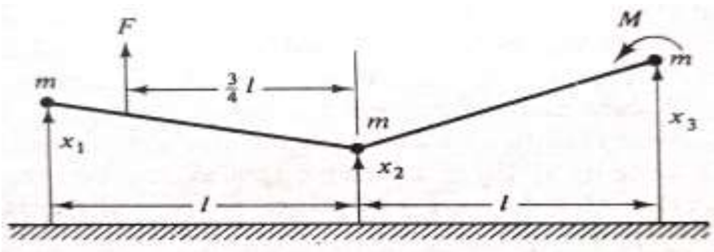
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) A particle of mass m is suspended by a massless wire of length $r = a + b\cos\omega t$ $\left(a > b > 0\right)$ to form a spherical pendulum. Find the equations of motion.

(OR)

b) Three particles are connected by two rigid rods having a joint between them to form the system shown in figure below. A vertical force F and a moment M are applied as shown.



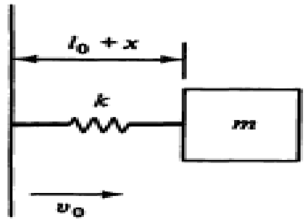
The configuration of the system is given by the ordinary coordinates $\left(x_1, x_2, x_3\right)$ or by the generalized coordinates $\left(q_1, q_2, q_3\right)$, where $x_1 = q_1 + q_2 + \frac{1}{2}q_3$, $x_2 = q_1 - q_3$, $x_3 = q_1 - q_2 + \frac{1}{2}q_3$. Find the generalized forces Q_1, Q_2 and Q_3. Assume small motions.

(CONTD…..3)

17.a) A small tube , bent in the form of a circle of radius r, rotates about a vertical diameter with a constant angular velocity ω. A particle of mass m can slide without friction inside the tube. At any given time, the configuration of the system is specified by the angle θ which is measured from the upward vertical to the line connecting the center O and the particle. Find the Jacobi integral.

(OR)

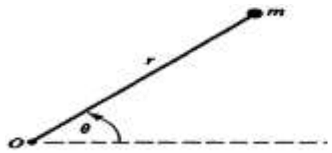
b) Suppose a mass-spring system is attached to a frame which is translating with a uniform velocity v_0 as shown in figure below. Let l_0 be the unstressed spring length and use the elongation x as the generalized coordinate. Find the Jacobi integral for the system.



18. a) Find a curve $y(x)$ between the orgin O and the point (x_1, y_1) such that a particle starting from rest at O, and sliding down the curve without friction under the influence of a uniform gravitational field, will reach the end of the curve in a minimum time.

(OR)

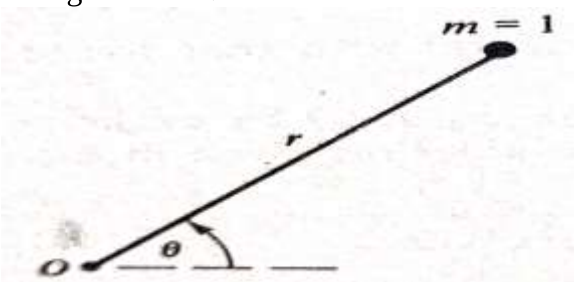
b) A particle of mass m is attracted to a fixed point O by an inverse square force, that is, $F_r = -\frac{\mu m}{r^2}$ where μ is the gravitational coefficient. Using the plane polar coordinates (r, θ) to describe the position of the particle (Fig.), find the equations of motion.



19. a) Derive the Hamilton –Jacobi equation.

(OR)

b) Suppose a particle of unit mass is attracted by an inverse-square gravitational force to a fixed point O in figure below.



Use Hamilton Jacobi method to obtain the position of the particle given in terms of polar coordinates (r, θ) measured in the plane of the orbit.

20.a) Consider the transformation $Q = \log \frac{\sin p}{q}$, $P = q \cot p$ Obtain the four major types of generating functions associated with this transformation.

(OR)

b) Consider a system having n degrees of freedom. Obtain a generating function for the resultant transformation equivalent to a sequence of two simple transformations, namely, a translation followed by a rotation.

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