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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2022 ONLY)

22PMS205

REG.NO. :

N.G.M. COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY-2023

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: II **TIME: 3 HOURS**

LINEAR ALGEBRA

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. (K1)

MULTIPLE CHOICE QUESTIONS.

1. Characteristics values are otherwise called ____________ roots.
   a) real b) complex c) latent d) imaginary

2. If V is a vector space, a ____________ of V is a linear operator E on V such that $E^2 = E$
   a) operator b) projections c) linear d) space

3. Let W be a proper T invariant subspace , then $W \cap Z(\alpha;T) =$ ____________.
   a) {4} b) {0} c) {2} d) {1}

4. In Jordan form, N is a ____________ linear operator on the finite dimensional space. V.
   a) Nilpotent b) normal c) adjoint d) self adjoint

5. The rank of the bilinear form is equal to the rank of the matrix of the form in any ordered_____.
   a) zero b) scalar c) vector d) basis

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define Minimal polynomial for T.
7. Define Nilpotent Operator N.
8. Define T annihilator of α.
9. What are considered as elementary row transformation ?
10. Define a Bilinear form.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Prove that the similar matrices have the same characteristic polynomial.

(OR)

b) Let T be a linear operator in an n dimensional vector space V, then prove that the characteristics and minimal polynomial for T have the same roots, except for the multiplicities.

12. a) Let V be a finite –dimensional vector space. Let $W_1, W_2, ..., Wk$ be subspaces of V and let $W = W_1 + W_2 + \ldots\ldots, W_k$. Show that the following are equivalent

i) $W_1, W_2, \ldots W_k$ are independent

ii) For each j, $2 \le j \le k$ we have $W_i \cap (W_1 + W_2 + \ldots\ldots, W_{i-1}) = \{0\}$

iii) If $\mathfrak{B}_I$ is an ordered basis for $W_i, 1 \le i \le k$ then the basis $\mathfrak{B} = (\mathfrak{B}_1, \mathfrak{B}_2, \ldots \mathfrak{B}_n)$is an ordered basis for W.

(OR)

(CONTD……2)

b) Let T be a linear operator on the finite dimensional vector space V over the field F. Suppose that the minimal polynomial for T decompose over F into a product of linear polynomials. Prove that there is a diagnosable operator D on V and a Nilpotent Operator N on V such that
(i) T = D+N (ii) DN = ND. Prove that D and N are uniquely determined by (i) and (ii)

13. a) If U is a linear operator on the finite - dimensional space W, then U has a cyclic vector if and only if there is some ordered basis for W in which U is represented by the companion matrix of the minimal polynomial for U. Prove it.

(OR)

b) Let F be a field and let B be an nXn matrix over F. Show that B is similar over the field F to one and only one matrix which is in rational form.

14. a) Find the characteristic polynomial of A = $\begin{bmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{bmatrix}$ and also find (A-2I)(A+I)

(OR)

b) Let M and N be equivalent m X n matrices with entries in the polynomial algebra F[X], prove that δ_k(M) = δ_k(N) , $1 \leq k \leq$ min (m,n)

15. a) Find all bilinear forms on the space F^2, where F is a field.

(OR)

b) Let V be a finite-dimensional vector space over a field of characteristic zero, and let f be a symmetric bilinear form on V. Prove that there is an ordered basis for V in which f is represented by a diagonal matrix.

SECTION – C (5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a) Find the characteristics polynomial for A = $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ and also find the dimension of the space of characteristics vectors associated with the characteristics values?

(OR)

b) State and prove Cayley-Hamilton theorem.

17.a) Let T be a linear operator on a vector space V, and $W_1, W_2, \ldots W_R$ are subspaces of Vand $E_1, E_2, \ldots E_R$ are projection on V. Prove that a necessary and sufficient condition for each subspace $W_i (i = 1, 2, \ldots R)$ to be invariant under T is that T commutes with each projection.

(OR)

b) State and prove the primary Decomposition Theorem.

18. a) State and prove Cyclic Decomposition Theorem.

(OR)

b) State and prove Generalized Cayley-Hamilton theorem.

19. a) Let M be matrix in $F[x]^{m,n}$ which has some non-zero entry in its first column, and let p be the greatest common divisor of the entries in column 1 of M. Then M is row- equivalent to a matrix N which has $\begin{bmatrix} p \\ 0 \\ ; \\ \mathbf{0} \end{bmatrix}$ as its first column. Prove it.

(OR)

b) If M is an m X n matrix with entries in the polynomial algebra F[X], prove that M is equivalent to a matrix N which is in normal form.

20.a) Let f be a bilinear form on the finite dimensional vector space V. Let L_f and R_f be the linear transformation from V into V^* defined by($L_{f\alpha}$)(β) = f(α,β) = ($R_{f\beta}$) (α). Prove that rank (L_f) = rank (R_f)

(OR)

b) Let V be an n-dimensional vector space over the field of real numbers, and let f be a symmetric bilinear form on V which has rank r. Then there is an ordered basis { β_1, β_2, … β_n} for V in which the matrix f is diagonal and such that $f(\beta_j , \beta_j) = \pm 1$, j= 1,2, ….. r. Furthermore, the number of basis vector β_j for which $f(\beta_j , \beta_j) = 1$, is independent of the choice of basis. Prove it.

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