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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2021 ONLY)

21PMS417

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY - 2023

COURSE NAME: M.Sc.-MATHEMATICS | **MAXIMUM MARKS: 70**

SEMESTER: IV | **TIME : 3 HOURS**

ALGEBRAIC TOPOLOGY

SECTION – A (10 X 1 = 10 MARKS)

ANSWER ALL THE QUESTIONS.
MULTIPLE CHOICE QUESTIONS. K1

1. If $k : X \to Y$ is continuous map and if f and g are paths in X with $f(1) = g(0), then\ k \circ (f * g) =$ ______
   $a. (k \circ f) * (k \circ g)$ b. $(k * f) \circ (k * g)$ c. $k \circ f$ d. $k \circ g$

2. Let $p : E \to B$ be a map. If f is a continuous mapping of some space X into B, lifting of f is a map $\tilde{f} : X \to E$ such that $p \circ \tilde{f} =$ ______
   $a. \tilde{f}$ b. f c. p d. 0

3. If $f : B^2 \to B^2$ is continuous, then there exists a point $x \in B^2$such that $f(x) =$ ___
   a. x^2 b. –x c. x d. $\frac{1}{x}$

4. If $f : K \to L$ is a simplical map, then a simplical map $sdf : sd\ K \to sd\ L$ is defined as $(sd\ f)\ (\tilde{F}) =$ _______
   a. $f(F)$ b. $\widetilde{f(F)}$ c. $\tilde{f}(F)$ d. $f(\tilde{F})$

5. Two cycles representing the same homology class are said to be ______.
   a) a. Homologous b. Homology c. Chain d. Complex

ANSWER THE FOLLOWING IN ONE OR TWO SENTENCES. K2

6. Define nullhomotopic.
7. Define covering map.
8. Define homology equivalences.
9. Define simplical complex.
10. Define chain complex.

SECTION –B $(\mathbf{5 \times 4 = 20}\ \boldsymbol{MARKS})$

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. K3

11.a. Prove that the map $\hat{\alpha}$ is a group isomorphism.

(OR)

b. Prove that in a simply connected space X, any two paths having the same initial and final points are path homotopic.

(CONTD……2)

12. a. If $p : E \to B$ and $p' : E' \to B'$ are covering maps, then prove that $p \times p : E \times E' \to B \times B'$ is a covering map.

(OR)

b. Let $p : E \to \boldsymbol{B}$ be a covering map; $p(e_0) = b_{0.}$ If E is path connected, Prove that the lifting correspondence $\phi : \pi_1(B, b_0) \to p^{-1}(b_0)$ is sujective.

13.a. If A is a retract of X, show that the homomorphism of fundamental groups included by inclusion $j : A \to X$ is injective.

(OR)

b. Let $f : X \to Y$ be continuous ; $f(x_0) = y_{0.}$ If f is a homotopy equivalence, prove that $f_x : \pi_1(X, x_0) \to \pi_1(Y, y_0)$ is an isomorphism.

14.a. Prove that any triangulation (k, f) of a topological space X defines a regular CW-structure on X such that the $n^{th} - skeleton$ is given by $X^{(h)} = f\left(\left|k^{(n)}\right|\right), n \geq 0.$

(OR)

b. Prove that every map $f : S^n \to S^{n+k}, k \geq 1$ is null homotopic.

15.a. Prove that the composition $\Delta_n(X) \xrightarrow{\partial_n} \Delta_{n-1}(X) \xrightarrow{\partial_{n-1}} \Delta_{n-2}(X)$ is zero.

(OR)

b. If X is a point, then show that $H_0(X) \approx z.$

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS.
(16^{th} QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS.
(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Prove that the map $p: R \to S'$ given by the equation $p(x) = (cos2\pi x, sin2\pi x)$ is a covering map.

17. Prove that the relations $\simeq$ and $\simeq_p$ are equivalence relations.

18. Let $p : E \to B$ be a covering map, and $p(e_0) = b_0$. Prove that any path $f: [0,1] \to B,$ beginning at b_0 has a unique lifting to a path $\tilde{f}$ in E beginning at e_0.

19. State and prove the fundamental theorem of algebra.

20. Let k be a finite simplical complex and d is a linear metric on$|k|$. Prove that d is linear metric on $|sd\ k| = |k|$ and for any $F' \in sd\ k$ such that $F' \subset |F|$, where $F \in k$ is a q-simplex, the inequality of the diameters:$diam\ |F'| \leq \frac{q}{q+1}\ diam\ |F|$ holds.

21. If two maps $f, g : X \to Y$ are homotopic, then prove that they induce the same homomorphism $f_x = g_x : H_n(X) \to H_n(Y)$.

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